Manonmaniam Sundaranar University,

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Tirunelveli - 627 012 Tamilnadu, India

OPEN AND DISTANCE LEARNING (ODL) PROGRAMMES

(FOR THOSE WHO JOINED THE PROGRAMMES FROM THE ACADEMIC YEAR 2023–2024)

II YEAR

B.Sc. Physics

Course Material

Optics and Laser Physics

Prepared

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OPTICS AND LASER PHYSICS

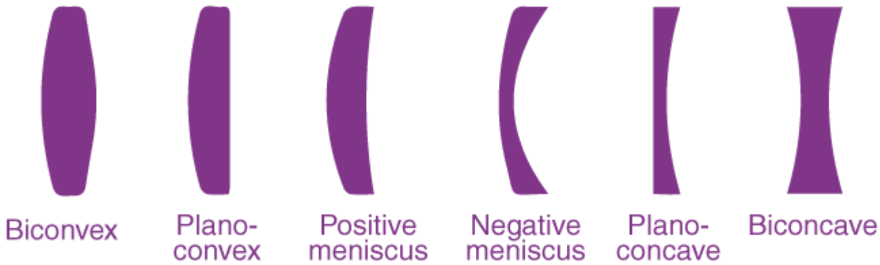
Unit	Details
I	**LENS AND PRISMS:** Lens: Lenses and its types – Equivalent focal length of two thin lenses in contact and separated by a distance – power of a lens. Aberrations: Spherical aberration, Methods of minimizing Spherical Aberration and chromatic aberrations. Prism:Dispersion by a prism, Angular dispersion and Dispersive power, Achromatic combination of prisms- Deviation without dispersion and Dispersion without deviation. Eyepieces:Eyepiece - Huygen‘s and Ramsden‘s eyepieces, construction and working – comparison
II	**INTERFERENCE:** Interference – Conditions – Theory of Interference - Fresnel‘s biprism – Experimental determination of the wavelength of light - Colours of thin films - Production of colours in thin films – Air wedge (Wedge-shaped film) – Newton‘s rings. Michelson‘s interferometer – Applications, (i) determination of the wavelength of a monochromatic source of light and (ii) determination of a thickness of a mica sheet.
III	**DIFFRACTION:** Fresnel and Fraunhofer diffraction - Fresnel‘s explanation of Rectilinear propagation of light - zone plate – action of zone plate for an incident spherical wave front – differences between a zone plate and a convex lens – diffraction pattern due to a straight edge – plane transmission diffraction grating– experiment to determine wavelengths.
IV	**POLARISATION:** Polarisation of light -double refraction – Nicol prism – Plane, circularly and elliptically polarized light –quarter wave plate – half wave plate – production and detection of circularly and elliptically polarized lights – Optical activity - Fresnel‘s explanation – Laurent half shade polarimeter – experiment to determine specific rotatory power.
V	**LASERS**: general principles of lasers – properties of lasers action – spontaneous and stimulated emission – population inversion – optical pumping – He-Ne laser (principle and working) – CO_2 laser (principle and working) – laser applications – holography and its applications.
TEXTBOOKS	1. Subrahmanyam. N, Brijlal and Avadhanulu. M.N, 2014, A textbook of optics, 25th Edition, S.Chandand Co. 2. Murugeshan. R and Kiruthiga Sivaprasath, 2014, Optics and Spectroscopy, 9th Edition, S.Chandand Co.
REFERENCE BOOKS	1. Sathyaprakash, 1990, Optics, VII edition, Ratan Prakashan Mandhir, New Delhi. 2. Ajoy Ghatak, 2009, Optics, 4th Edition, PHI Pvt Ltd, New Delhi. 3. Jenkins A.Francis and White, 2011, Fundamentals of Optics, 4th edition, McGraw Hill Inc., NewDelhi.

Introduction

A lens is a portion of a transparent medium bounded by two regular curved surfaces; or by one spherical surface and a plane surface. Spherical surfaces are easy to make therefore most lenses are made of spherical surfaces and have a wide range of curvature. Other transparent materials such as Quartz, used Silica and plastics are also used in making lenses. A single lens with two refracting surfaces is a simple lens.

Mirrors V_s Lens

The most apparent distinction between mirrors and lenses is: that mirrors reflect light rays (light bounces back) while light rays are refracted (pass-through) through a lens. A mirror will have only one focal point, which is in front of the mirror. A lens has two focal points, each on either side.



Pole

The centre of the spherical refracting surface of the lens is called the pole. It is the point where the principal axis meets the surface of the lens.

Optical Centre

The point on the principal axis at the centre of the lens is called the optical centre.

Centre of Curvature

A lens has two spherical surfaces; these two spherical surfaces form a part of a sphere. The centre of these spheres is known as the centre of curvature.

Principal Axis

The principal axis is an imaginary line passing through the centres of curvature and the pole.

Aperture

The area of the lens suitable for refraction is called aperture. The aperture of the lens is the effective diameter of its light-transmitting area.

Focus

Focus is the point onto which collimated light parallel to the axis is focused.

Focal Length

The focal length is the distance between the optical centre and the focal point or focus of the lens.

Power

The power of the lens is the reciprocal of its focal length. The SI unit of power is dioptre.

Types of Lenses

Today, there are different types of lenses available. Generally, they are categorised either as a simple lens or a compound lens.

A simple lens is a single piece of magnifying material, while a compound lens consists of a number of simple lenses arranged along a common axis. Optical aberrations are sometimes found in simple lenses, while this property is eliminated in a compound lens. Another advantage of the compound lens is that the magnification of the lens can be adjusted as per the user's requirements.

Non-spherical Lenses

Aspheric Lens

An aspheric lens is often known as a non-spherical lens. An aspheric lens is a lens whose surface is not part of a sphere or a cylinder. The complex surface of an aspheric lens reduces or eliminates optical aberrations as compared to a simple lens. A single aspheric lens can replace a combination of simple lenses resulting in a system with a much-reduced size.

Cylindrical Lens

Lenses that have a curvature along only one axis are classified as cylindrical lenses. Their main purpose is to convert laser diode elliptical light into a round beam or to focus light into a line. Motion picture anamorphic lenses are an example of such lenses.

Fresnel Lens

A Fresnel lens is a lens whose optical surface is divided into narrow rings. This allows the lens to be much thinner and lighter than conventional lenses.

Other Lenses

Lenticular lenses are a group of microlenses that are used in lenticular printing. These lenses produce images that have an illusion of depth.

A bifocal lens has two or more graduated focal lengths.

A gradient index lens is a lens with flat optical surfaces, while an axicon lens features a conical optical surface.

According to the shape and purpose of the lens, they are classified into two types:

- Concave lens
- Convex lens

Concave Lens

A concave lens is a type of lens with at least one side curved inwards. A concave lens with both sides curved inward is known as a biconcave lens. Concave lenses are diverging lenses, that is, they spread out light rays that have been refracted through them. They have the ability to diverge a parallel beam of light. For a concave lens, the edges are wider than the centre, or the centre is thinner than the edges. Concave lenses are used in spectacles in order to overcome myopia or short-sightedness.

A concave lens produces a smaller image for the viewer. The focal point of a concave lens is the point from which the light rays parallel to the axis seem to diverge after passing through the lens. The distance from the optical centre of the lens to the focal point is called the focal length of the lens.

The image formed in a concave lens has the following characteristics:

- Located on the object side of the lens
- Virtual image
- Upright image
- Small in size (i.e., smaller than the object)
- The image formed in a concave lens is always in between the focal point and the optical centre. The location of the object doesn’t affect the characteristics of the image formed.

Convex Lens

A convex lens is a lens with an outward curve. Unlike the concave lens, the thickness at the centre of a convex lens is more than the thickness at the edges of the lens. Convex lenses are converging lenses. They have the ability to converge a parallel beam of light into a point. This point is called the focal point of the convex lens, and the distance from the optical centre to the focal point is called the focal length. The focal point is on the opposite side of the lens from which the light rays originate.

A convex lens with one side flat is called a plano-convex lens. The lens found in the human eye is a prime example of a convex lens. Another common example of a convex lens is the magnifying glass that is used to correct hypermetropia or long-sightedness. Convex lenses are used in cameras as they focus the light and produce clear images. Convex lenses are also used in compound lenses which are employed in magnifying devices such as microscopes and telescopes.

Magnification

When a linear object is placed perpendicular to the principal axis of the lens, a linear image is formed perpendicular to the principal axis due to the refraction of the lens. The position, size and nature of the image formed depend on the position of the object with respect to the lens.

The ratio of the linear size of the image to the linear size of the object is called the magnification of the lens.

m = linear size of the image/linear size of the object

Optical Aberration

Optical aberration is a property of the lens that causes distortion or blurriness during image formation. When it occurs, the light is dispersed or spread out rather than being focused on a certain fixed point. Optical aberration is an undesirable property of lenses and can be eliminated by using a combination of lenses rather than using a single piece of lens.

There are several types of aberrations, such as spherical aberration, chromatic aberration and coma aberration, that can all affect image formation and quality.

Spherical Aberration

One of the primary reasons for the occurrence of spherical aberration is that the spherical surfaces of the lens are not the ideal shape, and as a result, the beams are parallel to but distant from the lens axis. This causes the blurring of the image. Spherical aberration can be corrected using a normal lens having the right surface curvatures for a particular application.

Coma Aberration

When an object is imaged off the optical axis of the lens, coma aberration takes place. Coma aberration can be eliminated by taking the curvature of the two lens surfaces that matches the application. Best form lenses can be used for it.

Chromatic Aberration

Chromatic aberration occurs due to dispersion. When it occurs, a lens fails to focus all colours on the same point. Chromatic aberration can be observed as colourful fringes around an image. It can be fixed using an achromatic doublet (or achromat).

There are other types of aberrations, and some of them include field curvature, astigmatism, and barrel and pincushion distortion.

Uses of Lens

- Used in a magnifying glass.
- Prosthetics for the correction of visual impairments.
- Attenuate light.
- Used in imaging systems.
- Used in radar systems.

Refraction through Lenses

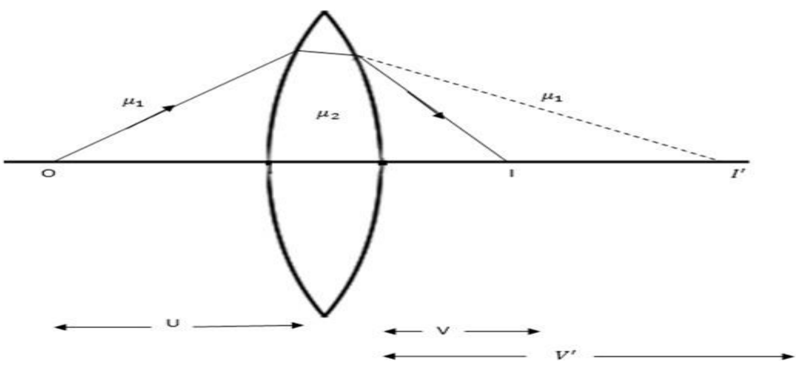
A lens is an image-forming device. It forms an image by refraction of light at its bounding surfaces, one spherical surface and a plane surface. The line joining the centers of curvature of the two spherical surfaces is known as the principal axis. If one of the surfaces is plane, the axis is a straight line normal to the surface drawn throughout the center of curvature of the other surface. A plane through the axis is called the principal section of the lens. Optical center of a lens is a point on the principal axis through which all the rays will pass, when the incident and emergent paths are parallel to each other.

Thin Lens

A thin lens is a lens with a thickness (distance along the optical axis between the two surfaces of the lens) that is negligible compared to the radii of curvature of the lens surfaces

Thin Lens Equation

Consider a thin lens of refractive index μ_2 placed in a medium of refractive index μ_1.



Let R_1 and R_2 be the radii of curvature of the two co-axial spherical surfaces and O is a point-object situated on the principal axis. An image I' is formed at a distance of v' from the pole of the first surface.

Then $\frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_{2-\mu_1}}{R_1}$ ---------------------------- (1)

If the distance from of the final image from the pole of the second surface is equal to v,

Then $\frac{\mu_1}{v} - \frac{\mu_2}{v'} = \frac{\mu_{2-\mu_1}}{R_2}$ ---------------------------- (2)

In this case the rays are passing from the medium of refractive index μ_2 to medium of refractive index μ_1

Adding equations (1) and (2)

$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ------------------------- (3)

Dividing equation 3 by μ_1,

$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ------------------------- (4)

If the lens is placed in air μ_1 = 1 and $\frac{\mu_2}{\mu_1}$ = μ, where μ is the refractive index of material of the lens. Then equation (4) becomes,

$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ------------------------- (5)

Equation 5 is known as the thin lens equation. Equation of a thin lens is

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

If the object is at infinity, the image is formed at the principal focus of the lens. When $u = \infty$, $1/u$ = 0 and $v = f$. then equation [1] becomes,

$$\frac{1}{f} - \frac{1}{\infty} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Equation (2) is known as the lens maker's formula, since it enables one to calculate f from the known properties of the lens. It can also be used to determine the values of R_1 and R_2 needed for a desired focal length of a lens of a given index of refraction. Magnification The magnification of a lens means how large or small, an object can be reproduced on the image plane. If an object of length X forms an image of length Y in the image, the magnification of the lens is defined to be Y/X. This is the lateral or transverse magnification (m) of a lens. If a small object of length du placed along the axis, produces an image of length dv along the axis, then Longitudinal magnification L = dv/du.

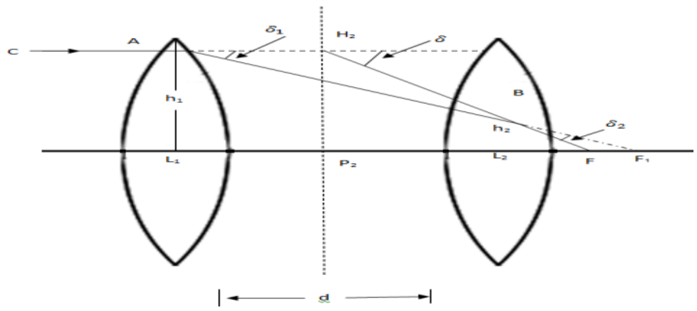
Power

The power of a lens is defined as the measure of its ability to produce convergence of a parallel beam of light. The unit in which the power of a lens is measured is called dioptre (D). Convex lens converges the light, so its power is taken as +ve. Concave diverge the light; hence its power is taken as –ve.

Mathematically, $power = \frac{1}{focal\ length\ in\ meter}$

Equivalent Focal Length of Two Thin Lenses

Let f_1 and f_2 be the focal lengths of two thin lenses L_1 and L_2 placed co-axially and separated by a distance d in air.



Let a ray IA of monochromatic light parallel to the common axis be incident on the first lens L_1 at a height h_1 above the axis. This ray, after refraction through the first lens, is directed towards F_1 which is the second principal focus of L_1. Then the deviation δ_1, produced by the first lens is given by,

$$\delta_1 = \frac{h_1}{f_1}$$

The emergent ray from the first lens is refracted by the second lens L_2 at a height h_2 and finally meets the axis at F. Since the incident ray IA is parallel to the principal axis and after refraction through the combination meets the axis at F, F must be the second principal focus of the combination. The deviation δ_2, produced by the second lens is given by

$$\delta_2 = \frac{h_2}{f_2}$$

The incident and the final emergent rays, when produced, intersect at E. It is clear that a single lens placed at P_2 will produce the same deviation as the two constituent lenses together. The lens of focal length P_2F placed at P_2 is termed as the equivalent lens which can replace the two lenses L_1 and L_2. The deviation produced by the equivalent lens is

$$\delta = \frac{h_1}{f}$$

where f is the focal length of the equivalent lens.

$$\delta = \delta_1 + \delta_2$$

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \text{----------------------------------} (1)$$

Δs AL_1F_1 and BL_2F_1 are similar

$\frac{AL_1}{L_1F_1} = \frac{BL_2}{L_2F_1}$ (or) $\frac{h_1}{f_1} = \frac{h_2}{f_{1-d}}$

$h_2 = \frac{h_1(f_1-d)}{f_1}$ -------------------------------------- (2)

Substituting this value of h_2 in equation (1)

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1-d)}{f_1 f_2}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{(f_1-d)}{f_1 f_2}$$

$$f = f = \frac{f_1 f_2}{f_1 + f_2 - d}$$

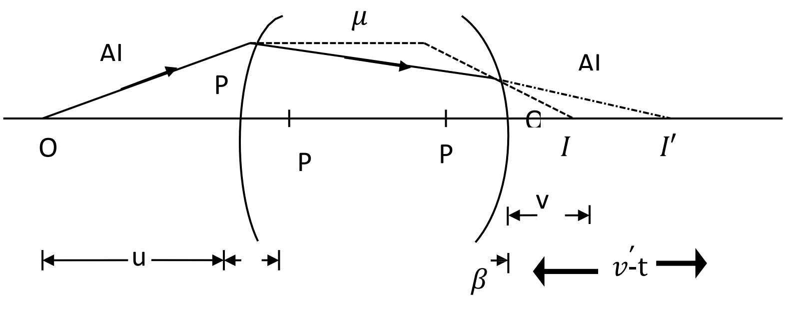
$$f = \frac{-f_1 f_2}{\Delta}$$

where$\Delta = d - (f_1 + f_2)$ and is known as the optical separation or optical interval between the two lenses. It is numerically equal to the distance between the second principal focus of the first lens and the first principal focus of the lens.

Thick Lens

A thick lens is a physically large lens having two spherical surfaces separated by a distance, which is not negligible in comparison to the radii of curvature of the spherical surfaces.

Thick Lens Formula Consider a lens of thickness t and of a refractive index μ, placed in air. The radii of curvature are R_1 and R_2.



A point object O is situated on the axis at a distance u from the first refracting surface and forms an image I′ at a distance v_1 from P.

$$\frac{\mu}{v_1} - \frac{1}{\mu} = \frac{\mu - 1}{R_1}$$

$$v_1 = \frac{R_1\mu\mu}{R_1 + \mu(\mu - 1)}$$

The image formed by the first surface acts as the object for the second surface and the final image is formed at I.

$$\frac{\frac{1}{\mu}}{v} - \frac{1}{v_1 - t} = \frac{\frac{1}{\mu} - 1}{R_2}$$

$$(v_1 - t) = \frac{\mu v R_2}{R_2 + v(\mu - 1)}$$

Substituting the value of v_1 from equation (1) in equation (2), we obtain

$$\frac{R_1\mu\mu}{R_1+\mu(\mu-1)} = \frac{\mu v R_2}{R_2+v(\mu-1)}$$

On simplification of the above expression, we obtain

$uv[\mu(\mu - 1)(R_1 - R_2) - (\mu - 1)^2 t] + u[\mu R_1R_2 - tR_2(\mu - 1)] +$

$v[-\mu R_1R_2 - tR_1(\mu - 1)] - tR_1R_2 = 0$ -------------------(3)

The equation is of the form

$uvA + uB + vC + D = 0$ ------------------- (4)

$$uv + u\frac{A}{B} + v\frac{C}{A} + \frac{D}{A} = 0$$

Where A, B, C, and D coefficients.

Let us take $V = v - \beta$ and $U = u - \alpha$ and the focal length=f

$\frac{1}{v-\beta} - \frac{1}{u-\alpha} = \frac{1}{f}$------------------------------------ (5)

Simplifying and rearranging the terms, we obtain

$uv + (-\beta - f)u + (-\alpha + f)v + (-\beta f + \alpha f + \alpha\beta) = 0$---------(6)

Comparing equation (5) and (6), we have

$-\beta - f = \frac{B}{A}$ -----------(7)

$-\alpha + f = \frac{C}{A}$ -----------(8)

And $-\beta f + \alpha f + \alpha\beta = \frac{D}{A}$ -----------(9)

From equation (7) to (9), we get

$$f^2 = \frac{D}{A} - \frac{BC}{A^2}$$

$$= \frac{AD-BC}{A^2}$$

Substituting the values of A, B, C and D in the above expression and after simplification, we get

$$f = \frac{\mu^2 R_1^2 R_2^2}{[\mu(\mu-1)(R_1-R_2)-(\mu-1)^2 t]^2}$$

$$f = \frac{\mu R_1 R_2}{[\mu(\mu-1)(R_1-R_2)-(\mu-1)^2 t]^2}$$

For a thin lens, t=0 and form equation (10) we see that

$$\frac{1}{f} = (\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

Power of Thick Lens

The power of a thick lens is given by

$$P = P_1 + P_2 - P_1 P_2 \,.\, t/\mu \text{ ----------(1)}$$

The power of the first refracting surface P$_1$ is $P_1 = (\mu - 1)/R_1$

and the power of the second refracting surface P$_2$ is $P_2 = -(\mu - 1)/R_2$

Substituting P$_1$ and P$_2$ in equation 1, we get

$$P = \frac{1}{f} = \frac{\mu-1}{R_1} - \frac{\mu-1}{R_2} + \frac{(\mu-1^2)}{R_1 R_2}\frac{t}{\mu}$$

Nodal Points

Nodal points are defined as a pair of conjugate points on the axis having unit positive angular magnification.

Defects of Images

The departure of real images from the ideal images, in respect of the actual size, shape and position are called aberrations. In other words, the failure of the lens to bring all rays from a point object to focus at the same point.

Aberrations

In an ideal optical system, all rays of light from a point in the object plane would converge to the same point in the image plane, forming a clear image. The influences which cause different rays to converge to different points are called aberrations.

Definition

The deviations from the actual size, shape and position of an image are called aberrations.

Types

The two types of aberrations are

i) Chromatic aberrations

ii) Monochromatic aberrations.

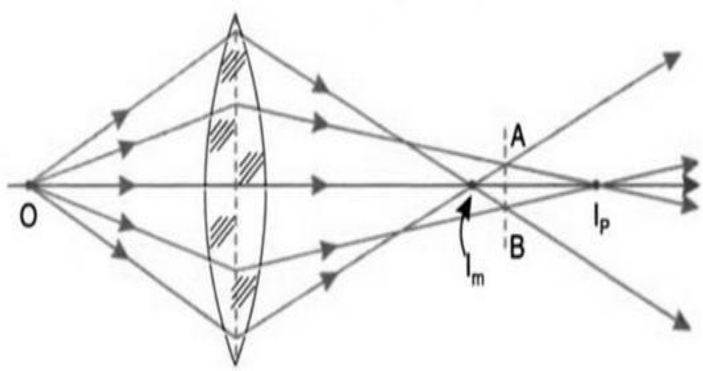
The aberrations produced by the variation of refractive index with wavelength of light are called chromatic aberrations. The aberrations caused even if monochromatic light is used are called monochromatic aberrations.

Monochromatic aberrations: Spherical aberration, coma, astigmatism, curvature of field and distortion are called monochromatic aberrations.

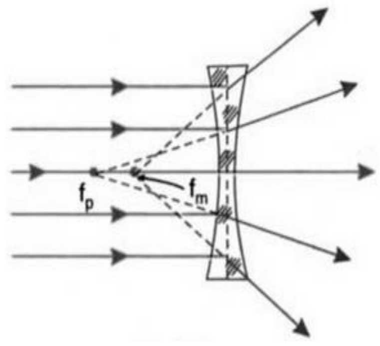
Spherical aberration due to thin lens

It is defined as failure of the lens to bring all rays from a point object situated on the axis to focus at the same point.

The presence of spherical aberration in the image formed by a single convex lens is illustrated in the figure.



O is a point object on the axis of the lens and I_p and I_m are the images formed by the paraxial and marginal rays respectively. The paraxial rays of light form the image at a longer distance from the lens than the marginal rays. The image is not sharp at any point on the axis. But at C, the image appears to be a circular patch of diameter AB. On either side of AB, the image patch has a larger diameter. The circular patch at C with diameter AB is the position of the best image and is called circle of least confusion. The distance between I_m and I_p is the longitudinal spherical aberration and the radius of the circle of least confusion is the lateral spherical aberration. For lenses made with spherical surfaces, rays which are parallel to the optic axis but at different distances from the optic axis, fail to converge to the same point. Thus, for an object point O on the axis, the image extends over the length $I_m I_p$. This aberration arises due to the fact that different annular zones of the lens have different focal length. The spherical aberration produced by a concave lens is shown below.



The spherical aberration produced by a convex lens is positive and by a concave lens is negative.

Methods of reducing Spherical Aberration

Spherical aberration can be minimised or eliminated by using

1. Stops
2. Crossed lens
3. Single Plano-convex lens
4. Two Plano-convex lens separated by a small distance
5. Combination of convex and concave lenses

i) Using stops

When the aperture of the lens is relatively large compared to the focal length of the lens, the cones of the rays of light refracted through the different zones of the lens surface are not brought to focus at the same point, resulting in spherical aberration. This can be minimised by using stops which reduce the effective lens aperture. Stops permit either the paraxial rays or marginal rays of light as shown.

The image appears less bright because the intensity of the incoming light is reduced by stops.

ii) Using crossed lens

The longitudinal spherical aberration produced by a thin lens for parallel incident beam is given by

$$X = \frac{\rho_2}{f_2}\frac{[k^2\mu^2 + k(\mu + 2\mu^2 + 2\mu^3 + \mu^3 - 2\mu^2 - 2]}{2\mu(\mu - 1^2)(1 - k^2)}$$

where x is the longitudinal spherical aberration, ρ is the radius of the lens aperture and f_2 is the second principal focal length.

$$K = \frac{R_1}{R_2}$$

where R_1 and R_2 are the radii of curvature. For given values of μ, f_2 and ρ, the condition for minimum spherical aberration is $\frac{dx}{dk} = 0$. Differentiating equation (1) and equating the result to zero $K = \frac{R_1}{R_2} = \mu(2\mu-1)-4\ \mu(2\mu+1)$ ------------(2)

from equation 2, if μ = 1.5, then k= - 1/6 . Thus the lens which produces minimum spherical aberration is biconcave. A lens whose R_1/R_2 = - 1/6 is called a crossed lens. Crossed lens can be used to minimize the spherical aberration.

iii) Using Plano- convex lenses

Plano- convex lenses are used in optical instrument so as to reduce the spherical aberration. When the curved surface of the lens faces the incident or emergent light whichever is more parallel to the axis, the spherical aberration is minimum. The spherical aberration in a crossed lens is only 8% less than that of a Plano-convex lens having the same focal length and radius of the lens aperture. Hence Plano-convex lens can also be used to minimise the spherical aberration.

iv) Using two Plano-convex lenses

Spherical aberration can be minimised by using two Plano-convex lenses separated by a small distance. The separation should be equal to the difference in their focal length. If f_1 and f_2 are the focal lengths of the two Plano-convex lenses, then the separation d = $f_1 - f_2$. The total deviation is equally shared by both the lenses and the spherical aberration is minimum.

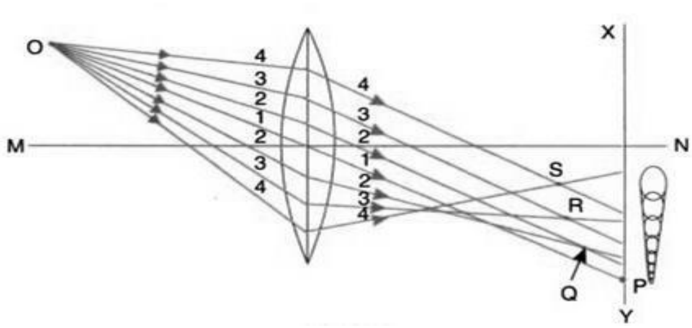
v) Combination of convex and concave lenses

Spherical aberration for a convex lens is +ve and that of a concave lens is –ve. By a suitable combination of convex and concave lenses, spherical aberration can be made minimum.

Coma

It is defined as failure of the lens to bring all rays from a point object not situated on the axis to focus at the same point.

The effect of rays from an object points not situated on the axis of the lens results in an aberration called coma. In spherical aberration, the point object is situated on the axis and the image is a circle of varying diameter along the axis. In case of comatic aberration, the point object is situated off the axis and the image are comet-shaped (circle of varying diameter normal the axis) and hence the name coma. The figure illustrates the presence of coma in the image due to a point object situated off the axis of the lens.



Rays of light getting refracted through the centre of the lens (ray 1) meet the screen XY at the point P. Rays 2,2; 3,3; etc., getting refracted through the outer zones of the lens come to focus at point Q, R, S etc., nearer the lens and on the screen overlapping circular patches of gradually increasing diameter are formed. The resultant image of the point is comet-shaped. Coma is the result of varying magnification for rays refracted through different zones of the lens. If the magnification of outer zone is lesser than inner zone, then coma is said to be +ve.

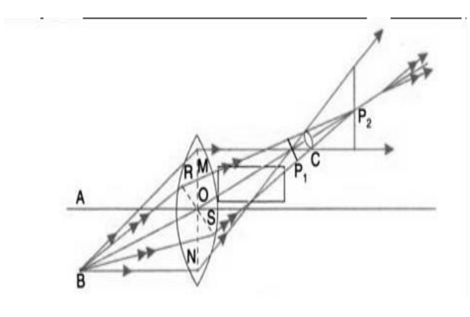
Elimination of coma:

Comatic aberration produced by a single lens can be corrected by properly choosing the radii of curvature of the lens surface. Coma can be altogether eliminated for a given pair of object and image points whereas spherical aberration cannot be completely corrected. Further, a lens corrected for coma will not be free from spherical aberration and the one corrected for spherical aberration will not be free from coma. Use of a stop or a diaphragm at the proper position eliminates coma.

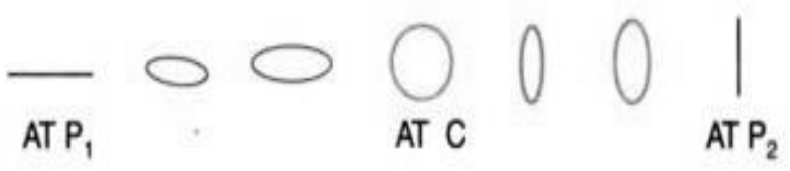
Astigmatism

Astigmatism is the aberration formed by a lens in the image of object points off the axis.

It is similar to coma (object is situated off the axis). However, in coma, the spreading of the image takes place in a plane perpendicular to the lens axis and in astigmatism the spreading takes place along the lens axis. Fig. illustrates the defect of astigmatism in the image of a point B.



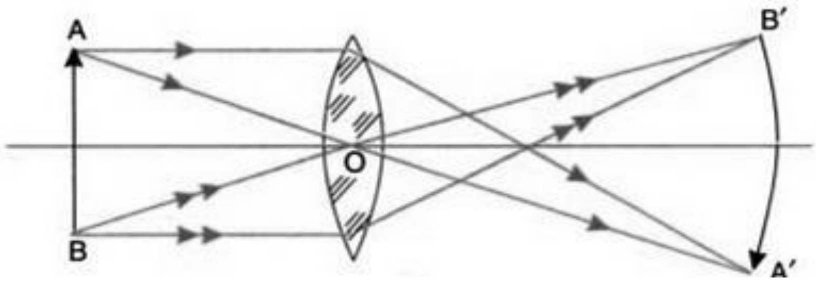
The cone of rays of light refracted through the tangential (vertical) plane BMN comes to focus at point P_1 nearer the lens and the cone of rays refracted through the sagittal (horizontal) plane BRS comes to focus at the point P_2 away from the lens. All rays pass through a horizontal line passing through P_1 called the primary image and also through a vertical line passing through P_2 called the secondary image. The refracted beam has an elliptical cross-section which ends to a horizontal line at P_1 and a vertical line P_2 as shown in the fig.



The cross section of the refracted beam is circular at some point between the primary and the secondary images and this is called the circle of least confusion. The focus of all the primary images of all points is called the primary image surface and locus of the secondary images gives the secondary image surface. If the primary image surface is to the left of the secondary image surface, astigmatism is said to be positive, otherwise negative. By using a convex and a concave lens of suitable focal lengths and separated by a distance, it is possible to minimise the astigmatic difference and such a lens combination is called an anastigmatic.

Curvature

The image of an extended plane object due to a single lens is not a flat one but will be a curved surface and this defect of a lens is known as curvature of field. Curvature of field causes a planar object to project a curved (non planar) image. It can be thought of arising from a "power error" for rays at a large angle. The central portion of the image nearer the axis is in focus but the outer regions of the image away from the axis are blurred. This is due to the fact that the paraxial focal length is greater than the marginal focal length. Fig. illustrates the presence of curvature of field in the image formed by a convex lens.



This aberration is present even if the aperture of the lens is reduced by a suitable stop.

Elimination: For a system of thin lenses, the curvature of the final image is given by

$$\frac{1}{R} = \Sigma \frac{1}{\mu_n f_n}$$

where R is the radius of curvature of the final image, μ_n and f_n are the refractive index and focal length of the nth lens. For the image to be flat, R must infinity. If two lenses are used which are placed in air, the condition for no curvature is

$$\frac{1}{\mu_1 f_1} + \frac{1}{\mu_2 f_2} = 0$$

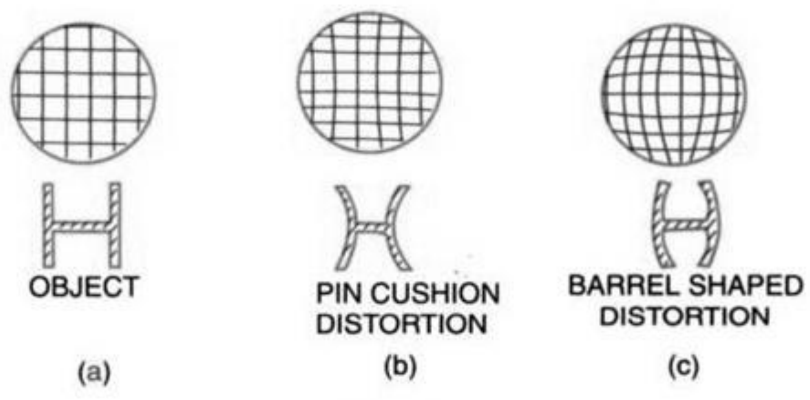
This is known as Petzwal's condition for no curvature. The above condition will be satisfied if the lenses are of opposite sign. If one of the lenses is convex the other must be concave.

Distortion

The variation in the magnification produced by a lens for different axial distances results in an aberration called distortion.

Distortion is of two types namely a) Pin – cushion distortion and b) Barrel shaped distortion.

In pin cushion distortion, the magnification increases with increasing axial distance and the image of an object appears as shown below in fig (b).



If the magnification decreases with increasing axial distance, it results in barrel shaped distortion and the image appears as shown in fig (c).

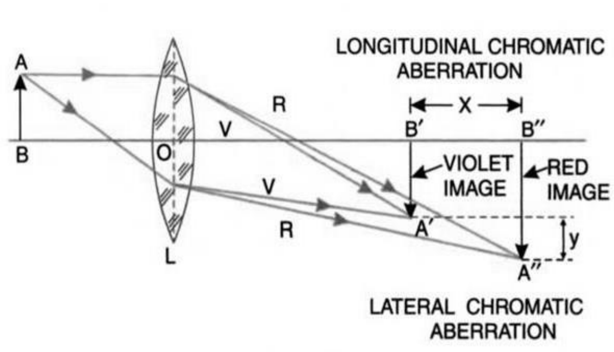
Elimination: If a stop is placed before the lens the distortion is barrel-shaped and if a stop is placed after the lens, the distortion in pin-cushion type. To eliminate distortion, a stop is placed in between two symmetrical lenses, so that the pin-cushion distortion produced by the first lens is compensated by the barrel-shaped distortion produced by the second lens.

Chromatic Aberration

A lens will not focus different colours in exactly the same place because the focal length depends on refraction and the refractive index of the material of a lens is different for different wavelengths of light. Hence the focal length of a lens is different for different wavelengths.

The index of refraction for blue light (short wavelengths) is larger than that of red light (long wavelengths). Further, as the magnification of the image is dependent on the focal length of a lens, the size of the image is different for different wavelengths (colours). The amount of chromatic aberration depends on the dispersion of the glass. It is defined as an aberration where a single lens produces a coloured image of an object illuminated by white light.

Chromatic aberration present in an image formed by a single lens is illustrated in the Fig.



The violet image is formed nearer the lens than the red image. The distance x measures the axial or longitudinal chromatic aberration and the distance y measures the lateral chromatic aberration.

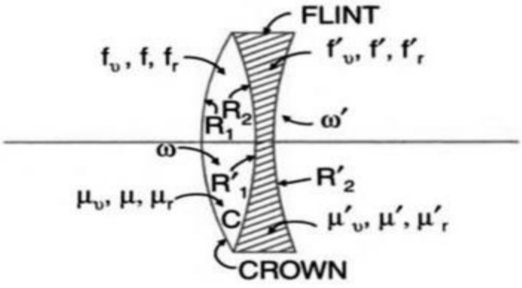
Elimination of this defect in a combination of lenses is called achromatism.

Condition for Achromatism

Chromatic aberration cannot be eliminated by using a single lens. Hence more than one lens is used. One way to minimize this aberration is to use glasses of different dispersion in a doublet or other combination.

Condition for achromatism of two lenses placed in contact:

The use of a strong positive lens made from a low dispersion glass like crown glass coupled with a weaker high dispersion glass like flint glass can correct the chromatic aberration for two colours, e.g., red and blue. A convex lens of crown glass and a concave lens of flint glass are placed in contact as shown.



Let μ_b, μ, μ_r and μ_b , μ', μ_r ' be the refractive indices for blue, yellow and red rays of light of the two materials. f_b, f, f_r and f_b', f' and f_r' are the corresponding focal lengths of the two lenses and ω and ω' are the dispersive powers for crown and flint glass respectively. The focal length of a lens is given by

$$\frac{1}{f} = (\mu - 1)\frac{1}{R_1} - \frac{1}{R_2}$$

$$\frac{1}{f_b} = (\mu_b - 1)\frac{1}{R_1} - \frac{1}{R_2}$$

$$\frac{1}{f_r} = (\mu_r - 1)\frac{1}{R_1} - \frac{1}{R_2}$$

$$\frac{1}{f'} = (\mu' - 1)\frac{1}{R_1'} - \frac{1}{R_2'}$$

$$\frac{1}{f_b'} = (\mu_b' - 1)\frac{1}{R_1'} - \frac{1}{R_2'}$$

$$\frac{1}{f_r'} = (\mu_r' - 1)\frac{1}{R_1'} - \frac{1}{R_2'}$$

$$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{(\mu - 1)f}$$

And $\frac{1}{R_1'} - \frac{1}{R_2'} = \frac{1}{(\mu' - 1)f}$

Substituting these values in equations

$$\frac{1}{f_b} = \frac{(\mu_b - 1)}{(\mu - 1)f}$$

$$\frac{1}{f_r} = \frac{(\mu_r - 1)}{(\mu - 1)f}$$

Let F_b and F_r be the focal lengths of the combination for blue and red rays of light.

$$\frac{1}{f_b} = \frac{(\mu_b - 1)}{(\mu - 1)f} + \frac{(\mu_b' - 1)}{(\mu' - 1)f}$$

$$\frac{1}{f_r} = \frac{(\mu_r - 1)}{(\mu - 1)f} + \frac{(\mu_r' - 1)}{(\mu' - 1)f}$$

For the combination to be achromatic, the focal lengths F_b and F_r must be equal. $F_b = F_r$ or $1/F_b = 1/F_r$

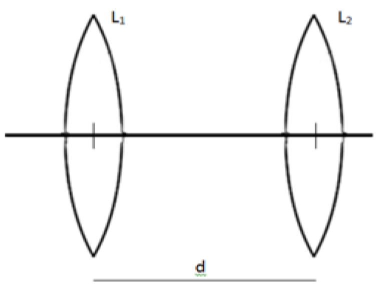
$$\frac{(\mu_b - 1)}{(\mu - 1)f} + \frac{(\mu_b' - 1)}{(\mu' - 1)f} = \frac{(\mu_r - 1)}{(\mu - 1)f} + \frac{(\mu_r' - 1)}{(\mu' - 1)f}$$

$$\frac{\omega}{f} + \frac{\omega'}{f'} = 0$$

$$f' = -f\frac{\omega'}{\omega}$$

Since ω and ω' are positive quantities, f' is negative if f is positive, i.e., if the crown glass is convex, then the flint glass lens is concave. Thus, the condition for achromatism is that the ratio of the dispersive powers of the materials of the lenses must be equal to ratio of the focal lengths of the two lenses.

ii) Condition for achromatism of two thin lenses separated by a finite distance Let f1 and f2 be the focal lengths of two lenses separated by a distance d as shown in fig.



The two lenses are made of the same material and μ, μband μr are the refractive indices for the mean rays, blue rays and red rays respectively for both the lenses. f_r, f_r' and f_b , f_b' are the focal lengths of the two lenses for red and blue rays of light. Then,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f_r'} - \frac{d}{f_r f_r'}$$

and $\frac{1}{F_b} = \frac{1}{f_b} + \frac{1}{f_b'} - \frac{d}{f_b f_b'}$

where F, F_r and F_b are the focal lengths of the combination for the mean rays, red rays and blue rays.

But

$\frac{1}{f_r} = \frac{(\mu_r-1)}{(\mu-1)f_1}$, $\frac{1}{f_r'} = \frac{(\mu_r-1)}{(\mu-1)f_2}$ and

$\frac{1}{f_b} = \frac{(\mu_b-1)}{(\mu-1)f_1}$, $\frac{1}{f_b'} = \frac{(\mu_b-1)}{(\mu-1)f_2}$

$\therefore \frac{1}{F_r} = \frac{(\mu_r-1)}{(\mu-1)f_1} + \frac{(\mu_r-1)}{(\mu-1)f_2} - \frac{(\mu_r-1)^2}{(\mu-1)^2}\frac{d}{f_1 f_2}$ and

$$\frac{1}{F_b} = \frac{(\mu_b-1)}{(\mu-1)f_1} + \frac{(\mu_b-1)}{(\mu-1)f_2} - \frac{(\mu_b-1)^2}{(\mu-1)^2}\frac{d}{f_1 f_2}$$

For the combination to be achromatic, the focal lengths F_b and F_r must be equal.

$F_b = F_r$ or $\frac{1}{F_b} = \frac{1}{F_r}$

$$\frac{(\mu_r-1)}{(\mu-1)f_1} + \frac{(\mu_r-1)}{(\mu-1)f_2} - \frac{(\mu_r-1)^2}{(\mu-1)^2}\frac{d}{f_1 f_2} = \frac{(\mu_b-1)}{(\mu-1)f_1} + \frac{(\mu_b-1)}{(\mu-1)f_2} - \frac{(\mu_b-1)^2}{(\mu-1)^2}\frac{d}{f_1 f_2}$$

Rearranging the above equation we get

$$\frac{(\mu_r-1)}{(\mu-1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) - \frac{(\mu_r-1)^2}{(\mu-1)^2}\frac{d}{f_1 f_2} = \frac{(\mu_b-1)}{(\mu-1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) - \frac{(\mu_b-1)^2}{(\mu-1)^2}\frac{d}{f_1 f_2}$$

Or $\frac{(\mu_b-\mu_r)}{(\mu-1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) = \frac{d}{(\mu-1)^2 f_1 f_2}[(\mu_b - 1)^2 - (\mu_r - 1)^2]$

$$= \frac{d}{(\mu-1)^2 f_1 f_2}[(\mu_b - \mu_r)(\mu_b + \mu_r - 2)]$$

$$= \frac{d(\mu_b-\mu_r)}{(\mu-1)^2 f_1 f_2}(2\mu - 2) \quad (\text{taking } \mu_b + \mu_r = 2\mu)$$

$$= \frac{d(\mu_b-\mu_r)}{(\mu-1)^2 f_1 f_2} 2(\mu - 1)$$

$$\therefore \frac{(\mu_b-\mu_r)}{(\mu-1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) = \frac{2d(\mu_b-\mu_r)}{(\mu-1)f_1 f_2}$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{2d}{f_1 f_2}$$

$$d = \frac{f_1 + f_2}{2}$$

Thus, the condition for achromatism of two thin co-axial lenses of same material separated by a distance is that the distance between the two lenses must be equal to the mean focal length of the two lenses.

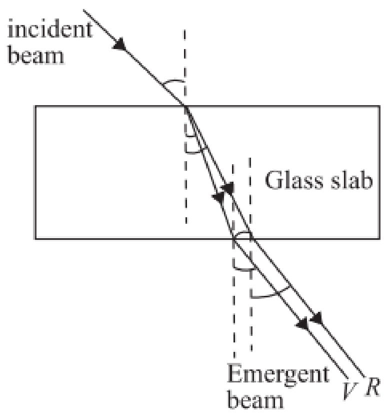
SUMMARY OF ABERRATIONS

Aberration	Character	Correction
Spherical aberration	Monochromatic, on- and off-axis, image blur	Bending, high index, aspheric, gradient index, doublet
Coma	Monochromatic, off-axis only, blur	Bending, spaced doublet with central stop
Oblique astigmatism	Monochromatic, off-axis blur	Spaced doublet with stop
Curvature of field	Monochromatic, off-axis	Spaced doublet

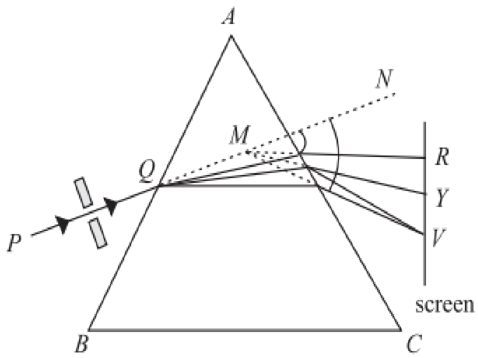
DISPERSION AND SCATTERING OF LIGHT

When a narrow beam of ordinary light is refracted by a prism, we see colour bands. This phenomenon has to be other than reflection or refraction. The splitting of white light into its constituent colours or wavelengths by a medium is called dispersion.

Dispersion through a Prism The separation of colours by a medium is not a sufficient condition to observe dispersion of light. These colours must be widely separated and should not mix up again after emerging from the dispersing medium. A glass slab (Fig.1) is not suitable for observing dispersion as the rays of the emergent beam are very close and parallel to the incident beam Newton used a prism to demonstrate dispersion of light.



White light from a slit falls on the face AB of the prism and light emerging from face AC is seen to split into different colours. Coloured patches can be seen on a screen. The face AC increases the separation between the rays refracted at the face AB. The incident white light PQ thus splits up into its component seven colours: Violet, indigo, blue, green, yellow, orange and red (VIBGYOR). The wavelengths travelling with different speeds are refracted through different angles and separated. This splitting of white light into component colours is known as dispersion. MR and MV correspond to the red and violet light respectively. These colours on the screen produce the spectrum. The bending of the original beam PQN along MR and MV etc. is known as deviation. The angle between the emergent ray and the incident ray is known as the angle of deviation. Thus δ_v and δ_r represent the angles of deviation for violet light and red light, respectively.



Angular dispersion

The difference between the angles of deviation for any two wavelengths (colours) is known as the angular dispersion for those wavelengths. The angular dispersion between the red and violet wavelengths is $\delta_V - \delta_R$. In the visible part of the spectrum, the wavelength of the yellow colour is nearly the average wavelength of the spectrum. The deviation for this colour δ_Y may, therefore, be taken as the average of all deviations.

The ratio of the angular dispersion to the mean deviation is taken as the dispersive power (ω) of the material of the prism :

$$\omega = \frac{\delta_v - \delta_R}{\delta_y}$$

We can express this result in terms of the refractive indices using Eqn.

$$\omega = \frac{\mu_v - \mu_R}{\mu_y - 1}$$

$$\omega = \frac{\Delta u}{\mu - 1}$$

EYE PIECE

- The eye-piece is one of the important parts of optical instruments.
- An optical instrument is required to produce a magnified image free from aberrations and a bright image covering a wide field of view.
- The single eye lens cannot produce such an image, so the extra lens called as field lens along with the eye lens is used in the eyepiece. The field lens and the eye lens together constitute an ocular or eyepiece. The two lenses are made and kept in such a way that their combination is achromatic and free from spherical aberrations.

Types of Eye Piece

Depending on the purpose of use, the eye-pieces are designed in different ways. There are mainly two types of eye-pieces:

(a) Positive eye-piece and

(b) Negative eye-piece.

Positive eye-piece:

Positive eye-pieces are provided with cross wires.

They are used for quantitative measurements like distance, angle etc. T

he cross-wires are placed outside and infront of field lens.

e.g. Ramsden eyepiece, Gauss eye-piece.

Negative eye-piece:

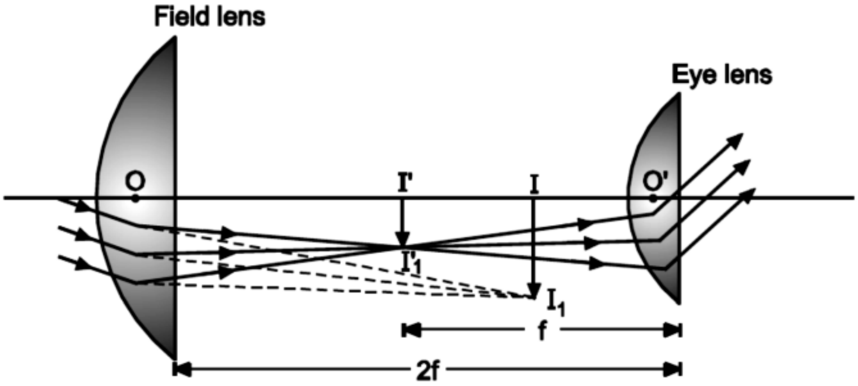
Cross wires are not used in negative eye-pieces.

They are used for structural studies like tissue structure, etc.

The cross wires are not used since they are not required in structural studies.

HUYGEN'S EYE-PIECE.

Huygen's eye-piece consists of two plano-convex lenses having focal length in the ratio 3 : 1. The distance between them is equal to the difference between the focal lengths. If f is the focal length of the eye-lens then 3f will be the focal length of the field lens.



II1 – Image of distant object formed by the objective in the absence of field lens. I'I'1 – Image formed by field lens. The image formed is at the focus of the eye-lens, hence image is seen at infinity. It is a negative type of eye-piece in which both chromatic and spherical aberrations are minimised.

Now, check the conditions for achromatic combination and for minimum spherical aberration.

(i) The condition for achromatic combination is that, the distance (d) between the lenses should be equal to the mean of their focal lengths. Here, the distance between the lenses is

3f-f = 2f

and the mean of focal lengths is,

3f+f/2 = 4f/2 = 2f

Hence from above equations the condition for achromatic combination is satisfied.

(ii) The condition for minimum spherical aberration is that two plano convex lenses facing towards the incident light must be kept apart with a distance equal to the difference in their focal lengths. Here, two plano convex lenses facing towards the incident light and the distance between the lenses is 3f – f = 2f. Hence, the condition for minimum spherical aberration is also satisfied.

EQUIVALENT FOCAL LENGTH (F):

The focal length of the combination for the eye-piece is,

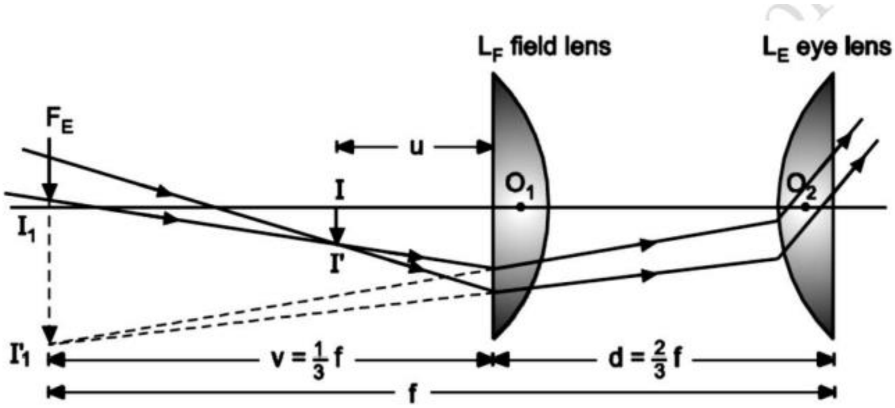
$$\frac{1}{f} = \frac{1}{f} + \frac{1}{3f} - \frac{d}{fX3f} = \frac{3}{2}f$$

Position of cross wires:

Cross wires cannot be used in the eye-piece. Because if the cross wires are to be used, they must be placed at I'I'1 between the field lens and eye lens. Then cross wire will be viewed through eye-lens only and the distant object will be viewed through both the lenses. Therefore, magnification of the cross wire and object will not be in the same proportion. Hence, the relative lengths of the images will also be not in the same proportion. Huygen's eye-piece is known as negative eye-piece, because the real inverted image (II1) formed by the objective is behind the field lens and it acts as a virtual object for the eye lens. Therefore, this eye-piece cannot be used to examine directly an object or real image formed by the objective.

RAMSDEN'S EYE-PIECE

Ramsden's eye-piece consists of two plano-convex lenses each of focal length separated by a distance equal to (2/3) f. The lenses are kept with their curved surfaces facing each other as shown in Fig. 2, thereby reducing spherical aberration.



The objective forms the image II' of the object at I. This serves as an object for the field lens. It forms its virtual image I1I'1 at the focus FE of the eyelens. Hence, distance between the eye lens and I1I'1 = f. The virtual image I1I'1 formed by the field lens acts as a virtual object for the eye lens. Hence, eye lens forms the final image of the object at infinity. Therefore, the rays emergent from the eye lens are mutually parallel.

Equivalent focal length (F):

If F is the equivalent focal length of the combination of field lens and eyelens then,

$$\frac{1}{f} = \frac{1}{f_1}+\frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{4}{3f} = \frac{3}{4}f$$

$$F=\frac{3}{4}f$$

Position of cross wires:

Hence, the cross-wires must be arranged at the distance of f/4 from the field lens outside the eye-piece. Since the cross-wires are arranged outside the eye-piece on the field lens side, Ramsden's eye-piece is called a positive eye-piece. The Ramsden's eye-piece has some chromatic aberration since the condition for achromatic combination is not satisfied. However, the spherical aberration is reduced because two plano convex lenses facing each other are used, but the condition for minimum spherical aberration is not satisfied.

Since $I_1O_2 = f \ and \ O_1O_2 = \frac{2}{3}f$

For field lens $I_1O_1 = \frac{1}{3}f \ and \ IO_1 = u$

Here $v = -\frac{f}{3}$ on sign convention

From equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$u = -\frac{f}{4}$$

Hence, the cross-wires must be arranged at the distance of f/4 from the field lens outside the eye-piece. Since the cross-wires are arranged outside the eye-piece on the field lens side, Ramsden's eye-piece is called a positive eye-piece. The Ramsden's eye-piece has some chromatic aberration since the condition for achromatic combination is not satisfied. However, the spherical aberration is reduced because two plano convex lenses facing each other are used, but the condition for minimum spherical aberration is not satisfied.

Comparison Between Huygen's eye-piece and Ramsden's eye-piece

Huygen's eye-piece	**Ramsden's eye-piece**
It is a negative eye-piece.	It is a positive eye-piece
The image formed by the objective lies between the field lens and the eye lens.	The image formed by the objective lies infront of the field lens.
Cross wires cannot be used.	Cross wires can be used.
It cannot be used for quantitative measurements in microscopes and telescopes.	It is used for quantitative measurements in microscopes and telescopes.

The condition for minimum spherical aberration is satisfied.	The condition for minimum spherical aberration is not satisfied.
The condition for achromatic combination is satisfied.	The condition for achromatic combination is not satisfied

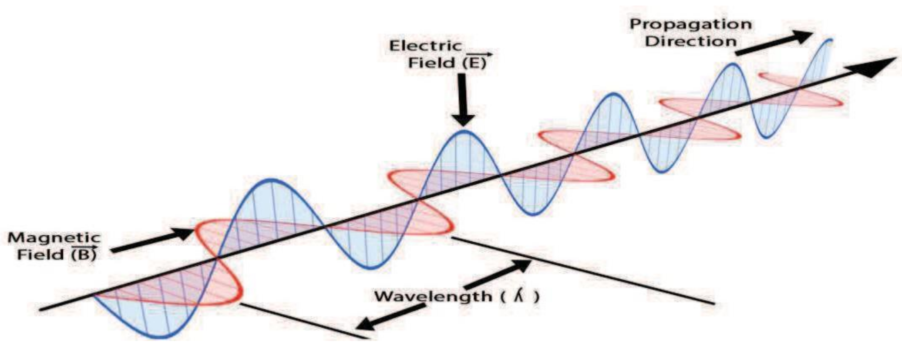
II-INTERFERENCE

In 1680 Huygens proposed the wave theory of light. But at that time, it was not clear about the nature of light wave, its speed and way of propagation. In 1801 Thomas Young performed an experiment called Young's double slit experiment and noticed that bright and dork fringes are formed which is called inference pattern. At that time it was a surprising phenomenon and is to be explained.

After the Maxwell's electromagnetic theory it was cleared that light is an electromagnetic wave. In physics, interference is a phenomenon in which two waves superimpose on each other to form a resultant wave of greater or lower or of equal amplitude. When such two waves travel in space under certain conditions the intensity or energy of waves are redistributed at certain points which is called interference of light and we observe bright and dark fringes.

WAVE NATURE OF LIGHT

Light wave is basically an electromagnetic wave. Electromagnetic wave consists of electric and magnetic field vectors. The directions of electric and magnetic vectors are perpendicular to direction of propagation as shown in the figure. The electric and magnetic vectors are denoted by E and H and vary with time.



In light, electric vectors (or magnetic vectors) vary in sinusoidal manner as shown in figure. Therefore the electric vectors can be given as

$$E = E_0 \sin (kz - \omega t)$$

Where E = Electric field vector, E_0 = maximum amplitude of field vector, k = wave number (= $2\pi/\lambda$), z = displacement along the direction of propagation (say z axis), ω = angular velocity and t = time.

Before understanding the interference, we should understand some terms and properties of light which are related to interference.

Monochromatic Light

The visible light is a continuous spectrum which consist a large number of wavelengths (approximately 3500Å to 7800Å). Every single wavelength (or frequency) of this continuous spectrum is called monochromatic light. However, the individual wavelengths are sufficiently close and indistinguishable. Some time we consider very narrow band of wave lengths as monochromatic light.

Ordinary light or white light, coming from sun, electric bulb, CFL, LED etc. consists a large number of wave lengths and hence non-monochromatic. But some specific sources like sodium lamp and helium neon laser emit monochromatic lights with wave lengths 589.3 nm and 632.8 nm respectively. It should be noted that sodium lamp, actually emits two spectral lines of wavelengths 589.0 nm and 589.6 nm which are very close together, and source is to be consider monochromatic.

Plane Wave

A plane wave is a wave whose wave front remains in a plane during the propagation of wave. In light wave, the maximum amplitude of electric vector E_0 remains constant and confined in a plane perpendicular to direction of propagation. Such type of wave called plane wave.

Polarized and Unpolarized Light

Light coming from many sources like sun, flame, incandescent lamp produces unpolarized light in which electric vector are oriented in all possible directions perpendicular to direction of propagation. But in polarized light electric vector are confined to only a single direction.

Phase Difference and Coherence

Wave is basically transportation of energy by mean of propagation of disturbance or vibrations. In wave motion through a medium, the particles of medium vibrate but in case of electromagnetic wave the electric or magnetic vectors vibrate form its equilibrium position.

The term phase describes the position and motion of vibration at any time. For example if $y = a \sin(\omega t + \theta)$ represents a wave, then the term $(\omega t + \theta)$ represents the phase of wave. The unit of phase is degree or radium. After completion of 360^0 or 2π, the cycle of wave or phase repeats.

Phase difference

If there are two waves have some frequency then the phase difference is the angle (or time) after which the one wave achieves the same position and phase as of first wave.

Coherence

If two or more waves of same frequencies are in same phase or have constant phase difference, those waves are called coherent wave.

Optical path and Geometric Path

Optical path length (OPL) denoted by Δ is the equivalents path length in the vacuum corresponding to a path length in a medium. Path length in a medium can be considered as geometric path length (*L*). Suppose a light wave travels a path length *L* in a medium of refractive index μ and velocity of light is *v* in this medium, then for a time period *t* the geometric path length *L* is given by

$$L = vt$$

In the same time interval *t*, the light wave travel a distance *Δ* in vacuum which is optical path length corresponding to length *L*. Then

$$\Delta = ct = c\frac{L}{v}$$

Where, c is the velocity of light in vacuum. or

$\Delta = \mu L$ or

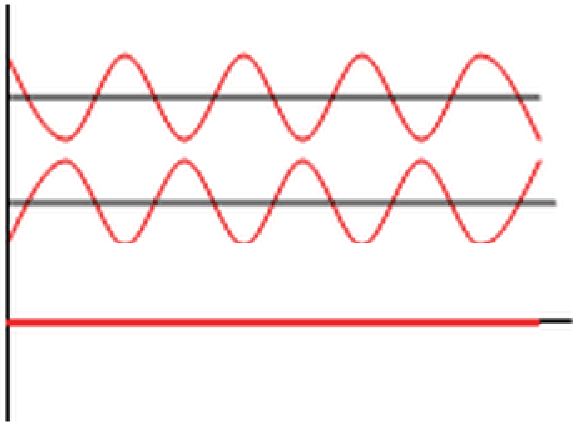
The Optical path length = μ × (Geometrical path length in a medium).

In case of interference, we always calculate optical path for simplification of understanding and mathematical calculations.

THEORY OF INTERFERENCE

When two light waves of some frequency, nearly same amplitude and having constant phase difference travel and overlap on each other, there is a modification in the intensity of light in the region of overlapping. This phenomenon is called interference.

The resultant wave depends on the phases or phase difference of waves. The modification in intensity or change in amplitude occurs due to principle of superposition. In certain points the two waves may be in same phase and at such point the amplitude of resultant wave will be sum of amplitude of individual waves. Thus, if the amplitudes of individual waves are a_1 and a_2 then the resultant amplitude will be $a = a_1 + a_2$. In this case, the intensity of resultant wave increases ($I \quad a^2$) and this phenomenon is called constructive interference. Corresponding to constructive interference we observe bright fringes.



On the other hand, at certain points the two waves may be in opposite phase as shown in above figure. In these points the resultant amplitude of waves will be sum of amplitude of individual waves with opposite directions. If the amplitudes of individual waves are a_1 and a_2 then the resultant amplitude will be $a = a_1 - a_2$ and the intensity of resultant wave will be minimum. This case is called destructive interference. Corresponding to such points we observe dark fringes. Above figure depicts two waves of opposite phase and their resultant.

Theory of Superposition

Let us consider two waves represented by $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin (\omega t + \delta)$.

According to Young's principle of superposition the resultant wave can be represented by

$$y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$$= a_1 \sin \omega t + a_2 (\sin \omega t \cos \delta + \cos \omega t \sin \delta)$$

$$= (a_1 + a_2 \cos \delta) \operatorname{Sin} \omega t + (a_2 \sin \delta) \cos \omega t \qquad \ldots\ldots. \quad (1)$$

Let $\qquad a_1 + a_2 \cos \delta = A \cos \emptyset \qquad \ldots\ldots. \quad (2)$

and $\qquad a_2 \sin \delta = A \sin \emptyset \qquad \ldots\ldots. \quad (3)$

Where A and Ø are new constants, then above equation becomes

$$y = A \cos \emptyset \sin \omega t + A \sin \emptyset \cos \omega t$$

or $y = A \sin(\omega t + \emptyset)$..(4)

This is the equation of the resultant wave. In this equation y represents displacement, A represents resultant amplitude, Ø is the phase difference.

From equation (2) and (3) we can determine the constant A and Ø. Squaring and adding the two equations, we get,

$$A^2 = a_1^2 + a_2^2 \cos^2 \delta + 2\, a_1 a_2 \cos \delta + a_2^2 \sin^2 \delta$$

$$A^2 = a_1^2 + a_2^2 + 2\, a_1 a_2 \cos \delta$$

On dividing equation, we obtain,

$$\frac{\sin\emptyset}{\cos\emptyset} = \tan\emptyset = \frac{a_{2\ \sin\partial}}{a_1 + a_2 \cos\delta}$$

Condition for Maxima or Bright Fringes

If $\cos \delta = +1$ then $\delta = 2n\pi$ where $n = 0, 1, 2, 3$……(positive integer numbers).

$$A^2 = a_1^2 + a^2 + 2a\, a = (a + a)^2$$

Intensity, $I = A^2 = (a_1 + a_2)^2$ -------------------------- (5)

Therefore, for $\delta = 2n\pi = 0, 2\pi, 4\pi$……, we observe bright fringes.

In term of path difference Δ

$$\Delta = \frac{\lambda}{2\pi n} \text{X phase difference} = \frac{\lambda}{2\pi n}\ 2n\pi$$

$$\Delta = n\lambda, \lambda, 2\lambda, 3\lambda \ldots\ldots etc$$

Condition for Minima or Dark Fringes

If $\cos \delta = -1$ or $\eth = (2n-1)\pi = \pi, 3\pi, 5\pi \ldots \ldots$

Then $A^2 = a_1^2 + a_2^2 - 2\, a_1 a_2 = (a_1 - a_2)^2$

Intensity, $I = A^2 = (a_1 - a_2)^2$

Therefore if phase difference between two waves is $\delta = (2n-1)\pi = 0, 3\pi, 5\pi \ldots$ etc. is the condition of minima or dark fringes.

Now path difference,

$\Delta = \frac{\lambda}{2\pi}$ X phase difference$=\frac{\lambda}{2\pi}$ $(2n-1)\pi = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$

Intensity Distribution

The intensity (I) of a wave can be given as $I = (½)\ \epsilon_o\ a^2$ where a is the amplitude of wave, and ϵ_0 is the permittivity of free space. If we consider two waves of amplitudes a_1 and a_2 then at the point of maxima

$$I_{max} = (a_1+a_2)^2 = a_1^2+a_2^2+2a_1a_2$$

If $a_1 = a_2 = a$ then $I = 4a^2$. Therefore, at maxima points the resultant intensity is more than the sum of intensities of individual waves.

Similarly, the intensity at points of minima

$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 = \ (a_1 \sim {a_2}^2)$

If $a_1 = a_2 = a$ then $I_{min}=0$. Thus, the intensity at minima points is less than the intensity of any wave.

The average intensity I_{av} is given as

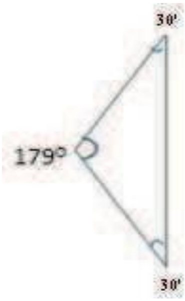
$$I_{av} = a_1^2 + a_2$$

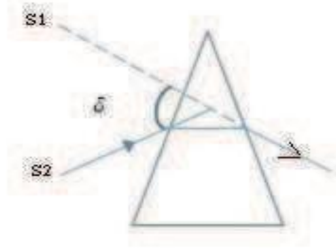
If $a_1 = a_2 = a$ then $I_{av} = 2a^2 = 2I$

Therefore, in interference pattern energy (intensity) $2a_1a_2$ is simply transferred from minima to maxima points. The net intensity (or average intensity) remains constant or conserved.

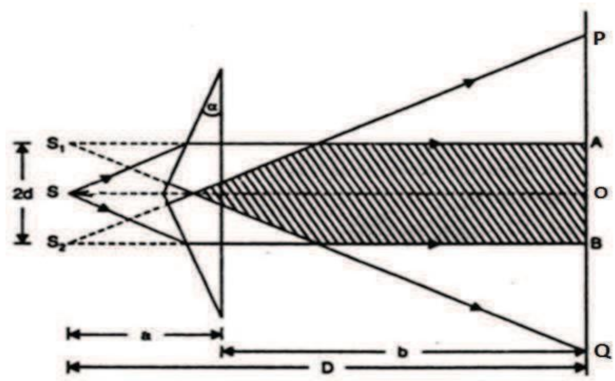
FRESNEL'S BIPRISM

Fresnel biprism consists of two acute angle prisms with their bases in contact. Generally, the angles are 179^0, $30'$ and $30'$ as shown in figure. The light coming from a source is allowed to fall symmetrically on a biprism as shown in figure. As we know, when a light beam is incident on a prism, the light is deviated from its original path through an angle called angle of deviations. Similarly in case of biprism, the light beam coming from source S, is appeared to be coming from S_1 and S_2 as shown in figure. Thus, we can say for prism S_1 and S_2 behave as virtual sources for the biprism.





In case of biprism, it can be considered that two cones of lights AS_1Q and BS_2P are coming from S_1 and S_2 and superimposed on each other and produce interference fringes in the region of superposition (between AB). The formation of interference fringes due to Fresnel's biprism is the same as due to Young's double slit experiment.



In this experiment point O is equidistance from both slits S_1 and S_2. If we consider distance between source and screen is D and separation between two slits S_1 and S_2 is 2d the fringe width can be given as

$$\omega = \frac{D\lambda}{2d}$$

The position of n^{th} bright fringe is given by $y_n = n\frac{D\lambda}{2d}$

Similarly, the position of n^{th} dark fringe is given by

$$y_n = \frac{2n-1}{2}\frac{D\lambda}{2d}$$

The wave length of the light source used in biprism experiment can be obtained by using above relation as

$$\lambda = \omega\frac{2d}{D}$$

Experimental Arrangement of Biprism Apparatus

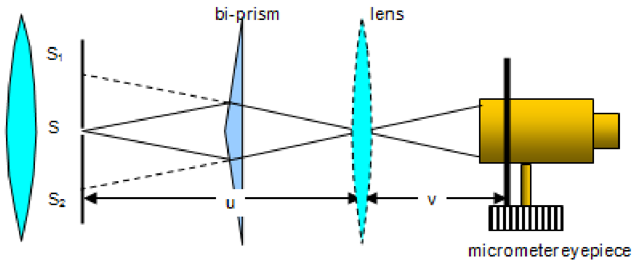
The experiment is performed on an optical bench as shown in figure. In this experiment we have an optical bench, which is an arrangement of two parallel metallic rods which are horizontal at same label. The rods or optical bench carry upright on which optical instruments are mounted. These uprights are movable on the rods. In the first uprights, we have a slit illuminated by a monochromatic light source S. The slit provides a linear monochromatic light to the biprism which is mounted on the second upright. The biprism is placed in such a way that its refracting edges parallel to the slit so that light falls symmetrically on the biprism. In third upright there is a concave lens for conversing the light coming from biprism. Finally, on forth upright a micrometer eyepiece is mounted in which interference fringes are observed.

For obtaining fringes, following adjustments are to be made.

(i) The optical bench is levelled with the help of spirit level.

(ii) Axis of slit is made parallel to edge of biprism.

(iii) The heights of all four uprights should be same so that line joining slit, biprism and micrometre should be parallel to optical bench.



Lateral Shift

If the eyepiece of micrometer is moved away from the biprism, and fringes shift either left or right of bench then it is called lateral shift. Simply, we can say the shift of fringes across the bench is called lateral shift. It indicates that the line joining the slit biprism and eyepiece is not parallel to the optical bench.

To remove the lateral shift, we put the eyepiece near the biprism and fix the vertical crosswire on any fringe. Now micrometer eyepiece is moved some distance away from biprism and direction of fringe shift is observed. Now biprism is moved in the direction opposite to the fringe shift so that vertical crosswise again reached on same fringe. We repeat this process again and again so that lateral shift removes compatibly.

Measurement of Wavelength of Light (λ) by Fresnel Biprism

By using the Fresnel biprism we can determine the wavelength of given source of light. For this purpose, we use the given light source in experimental arrangement. We adjust the apparatus for fringes are to be observed on the eyepiece. We measure the fringe width on apparatus and apply the formula for fringe width as

$$\omega = \frac{D\lambda}{2d}$$

$$\lambda = \omega\frac{2d}{D}$$

Fringe width ω can be measured with the help of micrometer on eyepiece. D is the distance between eyepiece and slit, and can be measured with the help of optical bend. The 2d is the distance between two virtual sources (S_1 and S_2) and cannot be measured directly with the help of any scale. We apply two methods for the measurement of distance 2d.

Magnification Method

To determine the distance 2d, we placed a convex lens of short focal length between biprism and screen. We find out a position L_1, of lens very near to biprism so that two sharp real images are obtained in the field of view of eyepiece. In figure the position of Lens L_1 is denoted by bold lines. In this position, we measure distance between two images d_1, with the help of micrometer of eyepiece.

For this position the magnification is given by

$$\frac{v}{u} = \frac{d_1}{2d}$$

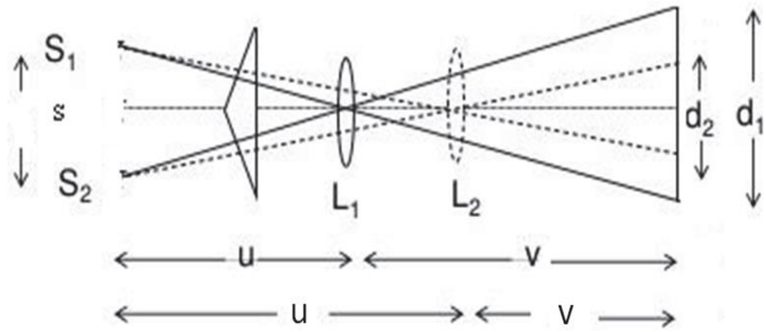
Now we move the lens some distance away from the biprism and obtain another position L_2 so that two sharp images are seen again in the field of view. We again measure the distance between two images, say d_2 with the help of micrometer of eyepiece.

In this case of position L_2 the magnification is given as

$$\frac{u}{v} = \frac{d_2}{2d}$$

$$2d = \sqrt{d_1 d_2}$$

By putting the value of d_1 and d_2 we can determine the value of 2d.



COLOURS OF THIN FILMS

When light coming from extended source is reflected by thin film of oil, mica, soap or coating etc., different colours are shown due to interference of light. For interference, the optical path difference is *Δ = 2µt Cos r = (2n+1) ß/2* for bright fringes. If thickness t is constant then for different wavelengths, angle of refraction r should be different. Therefore, different colours are observed at different angle of incident. Sometime different colours are over lopped on each other's and a mixed colour may be observed.

Production of Colors in thin films:

With monochromatic light alternate dark and bright interference fringes are obtained. With white light, the fringes obtained are colored. it is because the path difference $2\mu t \cos r - \lambda/2$ depends upon μ,t & r (i) Even if t and r kept constant, the path difference will change with μ & λ of light used. White light composed of various colors from violet to red. The path difference also changes due to reflection at denser medium by $\lambda/2$ as $\lambda_v \langle \lambda_R$

(ii) If the thickness of the film varies with uniformly, if at beginning it is thin, which will appear black. as path difference varies with thickness of the film, it appears different colors with white light.

(iii) If the angle of incidence changes, the angle of refraction is also changes, so that with white light, the film appears various colors when viewed from different directions.

AIR WEDGE

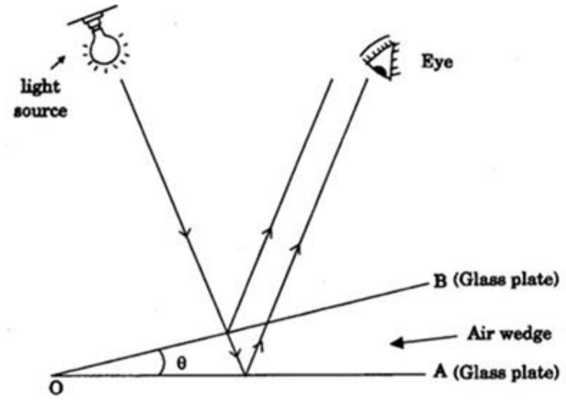
Air-wedge arrangement is used to find the thickness of a thin sheet or a wire. It is also used to test the planeness of the glass plate.

Definition

A wedge shaped (V-shaped) air film enclosed in between two glass plates is called air wedge.

Theory of air wedge experiment:

When two optically plane glass plates (*A* & *B*) are inclined at a very small angle 0, a **wedge-shaped thin air film** is formed between the surfaces as shown in fig. The thickness of the air film increases outwards from the line of contact 'O' of the glass plates.



The light rays from a monochromatic light source is made to fall perpendicularly on the film.

The incident rays of light are partially reflected from the upper surface of the air film and partially reflected from the lower surface of the air film.

These two reflected rays will interfere and a large number of **straight alternative bright and dark fringes** are formed.

If t is the thickness of the air film corresponding to the n^{th} dark band with wedge angle o at a distance of x metre from the edge of contact, then the path difference between the two reflected rays. Fringe width ω can be measured with the help of micrometer on eyepiece. D is the distance between eyepiece and slit, and can be measured with the help of optical bend. The 2d is the distance between two virtual sources (S_1 and S_2) and cannot be measured directly with the help of any scale. We apply two methods for the measurement of distance 2d.

$$2\mu t \cos r = n\lambda \text{ ------------- } (1)$$

For air film, refractive index of the film $\mu = 1$

$\cos r = 1$, since angle of incidence is very small, so angle of refraction is also very small ie., $r = 0$; $\cos \theta = 1$

$$\text{Now, } 2t = n\,\lambda \qquad \text{------------- } (2)$$

where λ - wavelength light

Since x is the distance of the n^{th} dark band from the edge of contact O,

$$\frac{t}{x} = tan\theta$$

$$\frac{t}{x} = \theta$$

$t\ = x\theta$ ------------------------ (3)

substituting equation (3) in equation (2), for the n^{th} dark band

$2x\theta = n\lambda$ -------------------(4)

Similarly, for the next dark band ie., $(n + 1)^{th}$ dark band

$2(x + \beta)\theta = (n + 1)\lambda$ ... (5)

where β is the fringe width

subtracting equation (4) from equation (5), we have

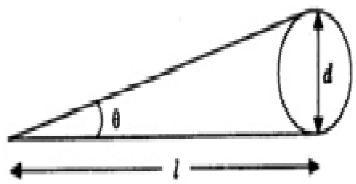
$$2\beta\theta = \lambda$$

or $$\boxed{\beta = \frac{\lambda}{2\theta}} \qquad \text{... (6)}$$

Thickness of a thin wire and very thin foil

The given wire whose thickness d is to be measured is placed inbetween the two glass plates to form a wedge-shaped air film.

Now if l is its distance from the edge of contact (length of the wedge), then from fig .



$$tan\theta = \frac{d}{l}$$

$$\theta = \frac{d}{l} \text{ ----------------------------(7)}$$

$$d = \frac{\lambda l}{2\beta}$$

Applications of air-wedge

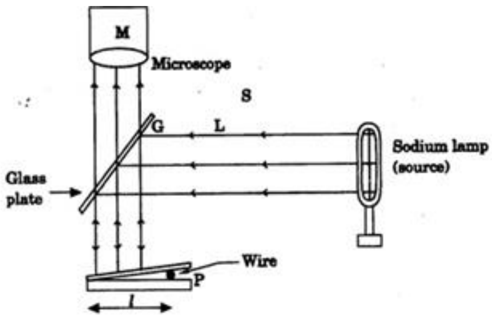
Determination of diameter (thickness) of a wire or thickness of a thin sheet of paper (Experiment)

An air wedge is formed by keeping two optically plane glass plates in contact along one of the edges and a thin wire near the other end, parallel to the contact edges of the glass plates.

Therefore, glass plater is inclined at a very small angle (one end of these two glass plates may be tied using a thread or a rubber band). This is called *air wedge arrangement*

Description

This arrangement is kept on the bed of the travelling microscope



A parallel beam of monochromatic light from a light source is reflected down on the air wedge by a glass plate kept inclined at an angle 45° to the horizontal.

Interference takes place between the light reflected at the top and bottom surfaces of the air film between the two glass plates.

Experiment

Interference pattern (Fig.) consisting of a series of bright and dark bands of equal width is viewed by a travelling microscope arranged above the airwedge.

Microscope is focussed on these fringes and the vertical cross wire is made to coincide with n^{th} bright band near the edge of contact of the glass plates.

The reading on the horizontal scale of the microscope is noted. The cross wire is made to coincide with successive 5th fringes (n +5, n + 10 ... n + 40) and the corresponding microscope readings are noted. The readings are recorded in the table.

From the table, the average fringe width β is determined. Using the microscope, the distance l between the edge of the contact and the wire is also measured.

Knowing the wavelength of the monochromatic light source, the thickness of the wire is found out using the formula.

$$d = \frac{\lambda l}{2\beta}$$

S.No	Order of the fringes	Microscope reading $x10^{-2}$m	Width of 10 fringes(m)	Band width β (m)
1.	n			
2.	n+5			
3.	n+10			

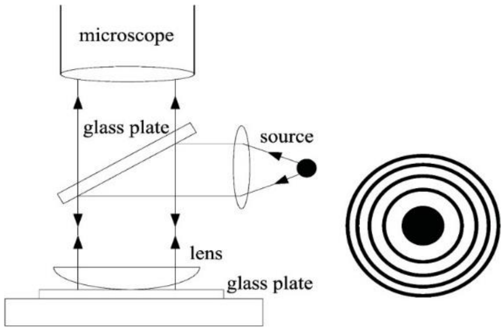
NEWTON'S RING EXPERIMENT

Newton's rings in a special case of wedge shaped film in which an air film is formed between a glass plate and a convex surface of lens. The thickness of air film is zero at the center and increases gradually towards the outside.

When a plano-convex lens of large focal length is placed on a plane glass plate, a thin air film is formed between the lower surface of plano-convex lens and upper surface of glass plate. When a monochromatic light falls on this film the light reflected from upper and lower surfaces of air film, and after interference of these rays, we get an inner dark spot surrounded by alternate bright and dark rings called Newton's rings. These rings are first observed by Newton and hence called Newton's rings.

Experimental Arrangement for Reflected Light

The experimental arrangement for Newton's rings experiment is shown in Figure. A beam of light from a monochromatic source S is made parallel by using a convex lens L. The parallel beam of light falls on a partially polished glass plate inclined at an angle of 45^0. The light falls on glass plate is partially reflected and partially transmitted. The reflected light normally falls on the plano-convex lens placed on plane glass plate.



(a) (b)

This light reflected from upper and lower surface of the air film form between plane glass plate and plano-convex lens. These rays interfere and rings are observed in the field of view. The figure shows the reflection of light form upper and lower surfaces of air film which are responsible for interference.

Formation of Bright and Dark Rings

As we know the interference occurs due to light reflected from upper and lower surface of air film form between glass plate and plano-convex lens. The air film can be considered as a special case of wedge shaped film. In this case, angle wedge is the angle made between the plan glass plate and tangent from line of contact to curved surface of plano convex lens as shown in figure.

The path difference between two interfering rays reflected by air film

$$\Delta = 2\mu t cos(r+\theta) - \frac{\lambda}{2}$$

where μ is the refractive index of the air film, t is the thickness of air film at the point of reflection (say point P) r is angle of refraction and ϴ is angle of wedge.

In this case the light normally falls on the plane convex lens for the angle of refraction r = 0. Further, as we use a lens of large focal length the angle of wedge ϴ is very small. So Cos (r+ϴ) = Cos ϴ = Cos 0^0 =1 and thus the path difference.

$$\Delta = 2\mu t - \frac{\lambda}{2}$$

At point of contact t = 0, therefore

$$\Delta = \frac{\lambda}{2}$$

Which is the condition of minima. Hence at centre or at point of contact there is a dark spot.

Condition of Bright Rings or Maxima

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n-1)\frac{\lambda}{2}$$

For Dark ring

$$2t = n\lambda$$

Thus corresponding to n = 1, 2, 3….. we observe first, second third…..etc. bright or dark rings. In Newton's rings experiment the locus of points of constant thickness is a circle therefore the fringes are circular rings.

In the above fig, from the property of the circle

$NP \times NQ = NO \times ND$

$r \times r = 2t \times (2R - t)$

$r^2 = 2Rt - t^2$

As t is small, t^2 is very small. So t^2 is neglected.

$r^2 = 2Rt^2$

$$t = \frac{D^2}{8R}$$

For Bright ring

$$D_n^2 = 2(2n-1)\lambda R$$

For dark Ring

$$D_n^2 = 4Rn\lambda$$

Determination of wave length of monochromatic light

$D^2 = 4Rn\lambda$

For n = m, $D^2 = 4Rm\lambda$

$$D_m^2 - D_n^2 = 4R\lambda(m-n)$$

$$\lambda = \frac{D_m^2 - D_n^2}{4R(m-n)}$$

This is the expression for wave length of monochromatic light.

Determination of refractive index of a liquid

The experimental set up as shown in fig. is used to find the refractive index of a liquid. To find the refractive index of a liquid, the plane glass plate and Plano convex lens set up is placed in a small metal container. The diameter of n^{th} and m^{th} dark rings are determined, when there is air between Plano convex lens and plane glass plate.

$$D_m^2 - D_n^2 = 4R\lambda(m-n)$$

Now the given liquid whose refractive index (μ) is to be introduced in to the space between Plano convex lens and plane glass plate without disturbing the experimental set up.

Then the diameters of Newton's rings are changed. Now the diameter of n^{th} and m^{th} dark rings are measured.

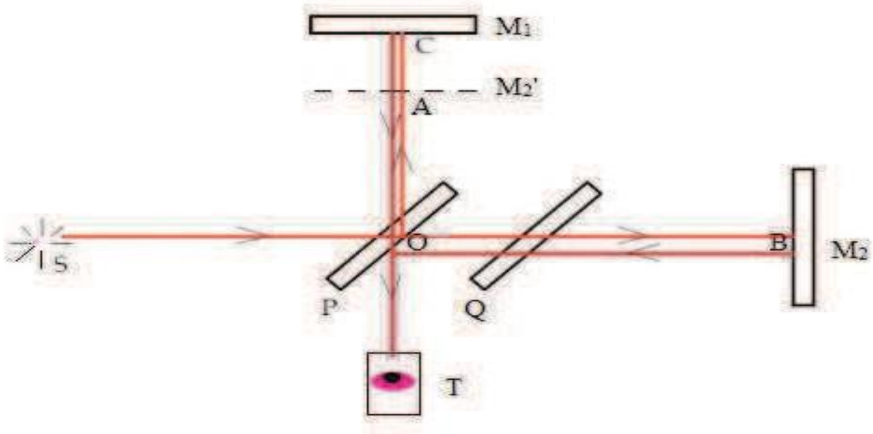
$$D_m^2 - D_n^2 = \frac{4R\lambda(m-n)}{\mu}$$

MICHELSON INTERFEROMETER

Michelson interferometer is a device used for the formation and study of interference fringes by a monochromatic light. In this apparatus, a beam of light coming from an extended source of light is divided into two parts, one is reflected part and another is refracted part after passing through a partially polished glass plate. These two beams are brought together after reflected from plane mirrors, and finally interference fringes are produced in the field of view.

Construction

The apparatus is shown in Figure. The main part of the apparatus is a half-silvered glass plate P, on which a beam of monochromatic light is incident. The plate P inclined at an angle 45^0 with incident light as shown in figur6.1, the incident light then divided into two parts, one is reflected part and another is transmitted part. The transmitted light is then passes through another glass plate Q which is of equal thickness as of P, and parallel to plate P, this pate Q is called compensating plate. The transmitted and reflected parts of light are normally incident on two mirrors M_2 and M_1 respectively. The mirror M_1 and M_2 are perpendicular to each other as shown in figure. The mirror M_1 is fixed in a carriage and can be moved to and fro with help of a screw and micro scale. Therefore, mirror M_1 is movable and the mirror M_2 is fixed. A telescope is also fixed as shown in figure. The light reflected from mirror M_1 and M_2 are superimposed and interference fringes are formed in the field of view.



Working

S is a source of monochromatic light; the light coming from this source is rendered parallel by mean of a convex lens L, and after passing through Less L the light falls on plate

P. Since plate P is partially polished, some part of light reflected back from P and going toward direction AC and incident on mirror M_1.

Similarly, the light transmitted from plate P passing through compensating plate Q and then incident on mirror M_2. The compensating plate is used to compensate the optical path travelled by transmitted light. The beam of light reflected by P, crosses plate P two times, for transmitted light this optical path is compensated by using plate Q in which the transmitted light crosses Q two time. Thus, by using compensating plate Q, the reflected and transmitted light travel equal optical path lengths.

Now the reflected light is incident on mirror M_1 and reflected back towards the telescope T. Similarly, the transmitted light incident normally on mirror M_2 and reflected back towards plate P, and at P some part of this light again reflected toward the telescope. Now in the direction of telescope we have two coherent beams of light reflected from mirror M_1 and M_2, and interference takes place and we observed interference pattern/beam in the field of view.

Formation of Fringes

Since the fringes are form by the light reflected from mirror M_1 (movable) and M_2 (fixed) and we can consider a virtual image of M_2 called M_2' in the field of view as shown in figure. Further we can consider the interference fringes are now formed due to light reflected from the surface of air film formed between mirror M_1 and M_2'. Now it is clear that the shapes of fringes are depend upon the inclination of mirror M_1 and M_2. Since M_2 fixed therefore the shape are depends upon the inclination of M_1. Since OA = OB, therefore the path difference between two rays are simply the path traveled in air film before reaching to telescope. If t is the thickness of air film, then path difference between light reflected from M_1 and M_2 is $2t$.

Condition for maxima

$$\Delta = 2t = n\lambda$$

$$2t = n\lambda$$

If the movable mirror M_1 moved by a distance x and we observed fringes shift of N fringes then

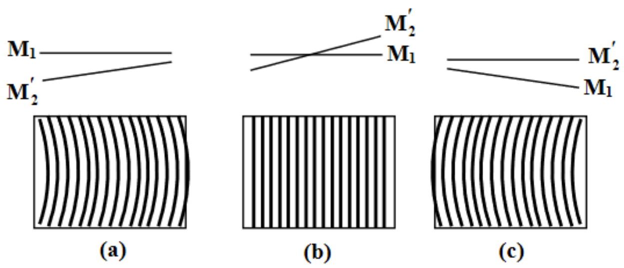
$$2(t + x) = (n + N)\,\lambda$$

or $$2x = N\lambda$$

or $$\lambda = \frac{2x}{N}$$

It is clear that if M_1 and M_2 are exactly perpendicular to each other, then M_1 and M_2' are parallel to each other and air film between M_1 and M_2' is of equal thickness in this case we observed fringes of equal inclination or Haidinger's fringes of circular shape. If, however, the two mirror M_1 and M_2 are not exactly perpendicular to each other than the shape of the air film formed between mirror M_1 and M_2' is of wedge shaped and the fringes are now of straight line parallel to the edge of wedge. These straight-line fringes are because of the focus of constant thickness in a wedge shape film is a straight line.

Thus, the shapes of fringes are depending on the inclination. The fringes are in general curved and convex toward the edge of wedge as shown in figure. These fringes are called localized fringes.



Determination of Difference of Wavelengths between Two Neighboring Wavelengths

Let us consider a source of light which emits two very close wavelengths. Sodium light is an example of such case. In sodium light, there are two wavelength D_1 *and* D_2 lines with wavelength λ_1 = 5890Å and λ_2 = 5896Å. By using Michelson interferometer, we can determine the difference between these two wavelengths. In this case first we adjust the aperture for circular fringes. We know that each wavelength produces its own ring spectrum. Now the mirror M_1 is moved in such a way that when the position of very bright fringes is obtained. In this position the bright fringes due to λ_1 coincident with the bright fringes due to λ_2 and we observe distinct fringes of order n.

Now the mirror M_1 is further moved to a very small displacement, and the fringes are disappeared. This case occurs when the maxima due to λ_1 coincident on minima due to λ_2. This is the position of minimum intensity or uniform illumination with no clear fringes. In this case we observed indistinct fringes of order (n+1). If we moved a distance x between such two points of most bright and most indistinct fringes then

$$2\,x = n\,\lambda_1 = (n+1)\,\lambda_2$$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2x}$$

If λ_1 and λ_2 are very close to each other then

$$\lambda_1.\lambda_2 = \lambda^2$$

Where λ is the mean value of λ_1 and λ_2

$$\lambda = \frac{\lambda_1+\lambda_2}{2}$$

$$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda_2}{2x}$$

Determination of Refractive Index of a Material

In Michelson interferometer, the two interfering beams of light travel in different directions, one is toward mirror M_1 and second one is toward mirror M_2. It is very easy to introduce a thin transparent sheet of a material of refractive index is and thickness t, in the path of one of the interfering beams of light. After introducing a sheet, the optical path of that beam increases by μt. Now the net increase in the path is $(\mu t - t)$. Since the beam crosses the sheet twice, the net path difference becomes $2(\mu t - t)$.

If n is the number of fringes be which the fringe system is displaced, then

$$2(\mu t - t) = n\,ß$$

or

$$2(\mu - 1)t = n\,ß$$

In experiment we first locate the central dark fringe by using while light. The cross wire of telescope is adjusted in such a way that the cross wire of telescope is adjusted on central dark fringes. Now the light is replaced by a monochromatic light of wavelength ß. Now a thin sheet is introduced into the path of one beam. The position of movable mirror M_1 is adjusted in such a way that the dark fringe is again coincide with the cross wire of telescope. We note the distance d through which the mirror is moved and count number of fringes displaced. By using the relation given below we can determine the thickness of sheet.

$$t = n ß / 2(\mu - 1)$$

Similarly, if we know the thickness, we can determine the refractive index of material.

$$2(\mu - 1)\, t = n ß$$

$$\mu = (n ß / 2t) + 1$$

Michelson Morley Experiment and Its Result

In classical mechanics it was assumed that the preferred medium for light propagation is ether which filled in all space uniformly. The ether is perfectly transparent medium of light and material bodies may pass in this medium without any resistance. Ether remains fixed in space and consider as absolute frame of reference. In the 19^{th} century this ether drag hypothesis of light was widely discuss.

Michelson interferometer was originally designed to verify the existence of hypothetical medium ether. The experiment performed to verify this hypothesis is called Michelson Morley experiment. In this experiment, it was assumed that the Michelson interferometer is moving along the earth direction of motion. Due to motion of apparatus with transmitted light are not same. Mathematically the path difference between two ray (transmitted and reflected) is lv^2/c^2 where l is distance between plate P and mirror M_1 and v is velocity of ether corresponding to this path difference there should be a fringe shift of $n = 0.37$. Thus if the apparatus is at rest and starts motion, there should be a fringe shift of $n = 0.37$. But it is not possible to make earth at rest. In this experiment we consider if the whole apparatus was turned by 90^o, the fringe shift should be observed.

The experiment was performed by many scientists, many times at different location on earth but fringe shift was not observed. This is called negative result of Michelson Morley experiment. The result shows the non existence of hypothetical medium of ether. After this experiment, a foundation of modern though way lay down which led to Einstein theory of relativity.

UNIT – III DIFFRACTION

In the preceding units we have read, that the interference phenomenon arises when two or more coherent light beams, obtained either by division of wavefront or by division of amplitude, meet each other. In this unit we shall discuss the interference effect of secondary wavelets originating from the same wavefront or from single aperture. This is called diffraction. The wave nature of light was further confirmed by the phenomenon of diffraction.

Diffraction refers to various phenomena which occur when a wave encounters an obstacle or a slit (or aperture). Since at the atomic level, physical objects have wave-like properties, they can also exhibit diffraction effects. The diffraction of light was first observed and characterized by an italian mathematician Francesco Maria Grimaldi. The word diffraction originated from Latin word ‘diffractus' which means ‘to break into pieces'. Thus he referred this phenomenon as breaking up of light into different directions. Isaac Newton attributed them to inflexion of light rays. James Gregory used a bird feather and observed the diffraction patterns. This was effectively the first diffraction grating to be discovered.
Augustin-Jean Fresnel did more studies and calculations of diffraction and thereby gave great support to the wave theory of light that had been advanced by Christiaan Huygens.

The effects of diffraction are often seen in everyday life. For example, the closely spaced tracks on a CD or DVD act as a diffraction grating for incident light and form a rainbow like pattern when seen at it. The hologram on a credit card is another example. Almost the same colourful pattern is formed due to the diffraction of light. A bright ring around a bright light source like the sun or the moon is because of the diffraction in the atmosphere by small particles.

DIFFRACTION OF LIGHT

As per the rules of geometric optics, the light should caste a well-defined and distinct shadow of an object placed in its path. If the direction of incidence of light is perpendicular to the length of obstacle, then due to its rectilinear propagation, the size of the image should be equal to the size of the object. No light should reach into the regions of shadow. The same thing happens with aperture. Light enters from the open region of aperture and reaches to the screen. When the direction of incidence is not normal to length of obstacle (or aperture), the size of image (or shadow) will be different from that of obstacle or aperture.

Figure 1

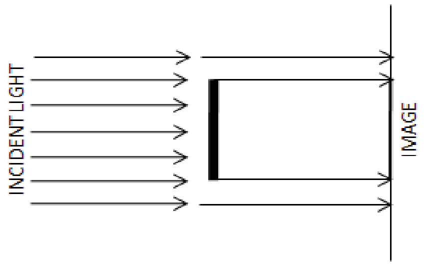
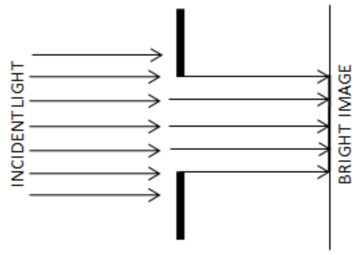


Figure 2



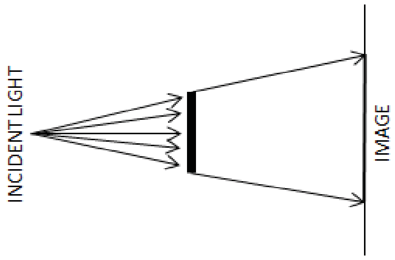


Figure 3

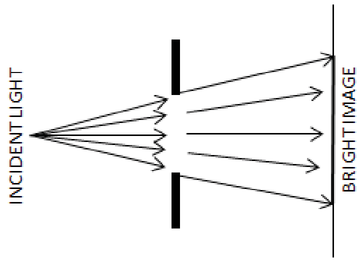
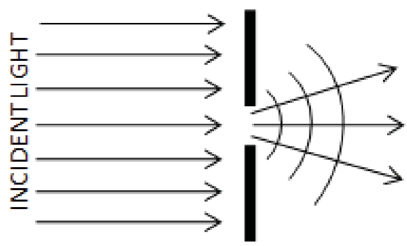


Figure 4

A very close and careful observation of light distribution reveals that there are dark and bright fringes near the edges. As the size of the aperture is decreased the fringes become more and more distinct. When the size of aperture becomes comparable to the wavelength of incident light the fringes become broad and practically cover the entire shadow region, so instead of a sharp shadow we obtain bright and dark fringes on the screen.



In simple language we can say that 'when the size of the opaque obstacle (or aperture) is small enough and is comparable to the wavelength of incident light, the light bends round the corners'. If the opening is much larger than the light's wavelength, the bending will be almost unnoticeable. The phenomenon of bending of light round the corner or edge and spreading into the geometrical shadow region of the obstacle (or aperture), placed in its path,is known as diffraction. The bending of light for a small slit is shown in figure. The formation of alternate bright and dark fringes, by the redistribution of light intensity, is called the diffraction pattern. The amount of bending depends on the relative size of the wavelength of light to the size of the opening.

Dominique Arago placed a small circular disc in between a point light source and screen and obtained almost a regular pattern of alternate dark and bright rings. There was a bright circular spot at the centre of this pattern. The formation of this kind of diffraction pattern could not be explained on the basis of rectilinear propagation of light. Thus wave theory of light was used to explain the bending of light into the regions of geometrical shadow.

FRESNEL AND FRAUNHOFER CLASSES OF DIFFRACTION

The diffraction phenomenon is usually divided into two classes; the Fresnel diffraction and Fraunhofer diffraction. Following are the main differences between these two types of diffractions.

(i) In Fresnel diffraction either the source of light or the screen or both are in general at finite distance from the diffracting element (obstacle or aperture) whereas in Fraunhofer diffraction both the source of light and the screen are at infinite distance from diffracting element.

(ii) In Fresnel diffraction no lenses are used for rendering the rays parallel or convergent therefore the incident wavefront is divergent either spherical or cylindrical. In Fraunhofer class of diffraction generally two convergent lenses are used; one to make the incoming light parallel and other to focus the parallel diffracted rays on the screen. The incident wavefront is, therefore, plane.

(iii) In Fresnel diffraction the phase of secondary wavelets is not the same at all points in the plane of aperture while converse is true for Fraunhofer diffraction.

(iv) Depending on the number of Fresnel's zones formed, the centre of the diffraction pattern may be either dark or bright in Fresnel diffraction but in Fraunhofer diffraction it is always bright for all paths parallel to the axis of lens.

(v) In Fresnel class of diffraction the lateral distances are important while in Fraunhofer diffraction the angular inclination plays important role in the formation of diffraction pattern.

(vi) In Fresnel diffraction the diffraction pattern formed is a projection of diffracting element modified by the diffracting effects and the geometry of the source and in Fraunhofer diffraction the diffraction pattern is the image of the source modified by the diffraction at diffracting element.

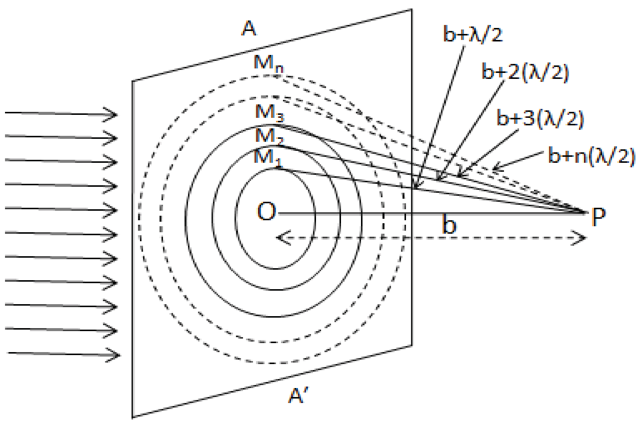
FRESNEL’S HALF PERIOD ZONES

According to Huygens principle each point on a wavefront acts as a source of secondary disturbance. When a wavefront is made to incident on a slit, most of it is obstructed by the slit. The small portion of the wavefront passed through the slit is, thus, equivalent to a string of coherent point sources. The intensity at any point on the screen may be obtained by suitably summing the intensities of wavelets originating from those point sources at the slit and superposing at that point of screen. Thus diffraction pattern is formed at screen due to the interference of secondary wavelets.

Since the coherent sources are located at different distances from any point on the screen, the waves reach that point with differing phases. Their superposition produces interference pattern with maxima and minima formation. Therefore, the diffraction of light is due to the superposition of waves from coherent sources of the same wavefront after the wavefront is obstructed by obstacle or aperture.

Construction of Zones

For the qualitative understanding of the diffraction pattern, Fresnel introduced the idea of half period zones. The wave-front originated from the source and striking the obstacle or aperture is divided into a number of the circular and the concentric zones. Zone is the small area on the plane wave-front with reference to the point of the observation such that all the waves from the area reach the point without any path difference. The paths of light rays from the successive zones differ by *ß/2*. Since path difference of *ß/2* corresponds to half time period, these zones are known as half period zones.



In order to understand the construction of half period zones taking a plane wavefront *AA′* and droping a perpendicular PO on the wavefront from an external point P. If the distance PO is *b* then taking *P* as a centre draw spheres of radii *b+ß/2, b+2(ß/2), b+3(ß/2) etc*. The spheres will cut the wavefront *AA′* in circles of radii OM_1, OM_2, OM_3 etc as shown in figure. The annular regions between two consecutive circles are called *half period zones*, e.g., the annular region between $(n-1)^{th}$ circle and n^{th} circle is called the n^{th} half period zone.

Radii and Area of Zones

From simple geometry the radius of n^{th} such circle, OM_n, can be written as

$$OM_n = r_n = \left[\left(b + n\frac{\lambda}{2}\right) - b^2\right]^{1/2}$$

$$= \sqrt{n\lambda b}$$

Here we have assumed $b >> ß$, which is true in most of the experiments using visible light. We have also assumed here that n is not a very large number. From expression given by equation, it is clear that the radii of half period zones are proportional to the square roots of natural numbers. Therefore, the radii of first, second, third etc. half period zones are $\sqrt{\lambda b}$, $\sqrt{2\lambda b}$, $\sqrt{3\lambda b}$ etc.,

With the help of equation, the area of n^{th} half period zone is given by

$$A_n = \pi r_n^2 - \pi r_{n-1}^2 = \pi[n\lambda b - (n-1)\lambda b] = \pi\lambda$$

Thus for $b >> ß$ and n not very large, the areas of half period zones are independent of n and are approximately equal for fixed value of $ß$ and b. The area of the zone may be varied by varying the wavelength of light used and the distance of the point from the wavefront.

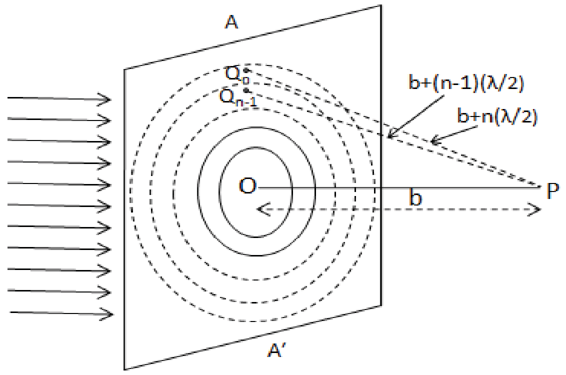
Resultant Amplitude at Point P

According to Fresnel the resultant amplitude at any point due to whole of the wavefront will be the combined effect of all the zones, while the amplitude produced by a particular zone is proportional to the area of the zone and inversely proportional to the distance of the zone from the point of consideration, P. This amplitude also varies with obliquity factor $\frac{1}{2}(1 + cos\theta)$ Where θ is the angle between the normal PO to the wavefront and the line QP.

Thus if u_n represents the amplitude produced by the secondary wavelets emanating from the n^{th} zone then we can write

$$u_n = \text{constant x } \frac{A_n}{Q_n\,P} \times \frac{(1+cos\theta)^n}{2}$$

Where θ_n is the value of θ for n^{th} zone. If we take infinitesimal areas around point Q_n in the n^{th} half period zone and around a corresponding similar point Q_{n-1} in $(n-1)^{th}$ half period zone as shown in the figure such that



$$Q_n P - Q_{n-1} P = ß/2 \;......................................(7.4)$$

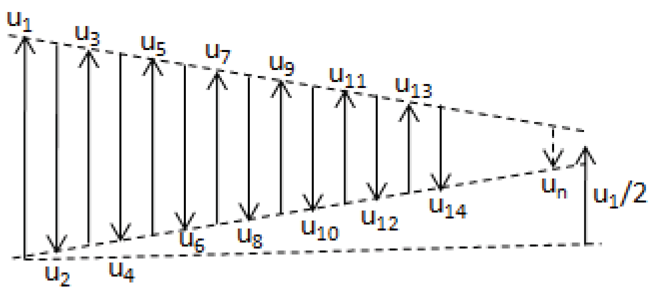
This path difference of $ß/2$ corresponds to a phase difference of π. Although the areas of the zones are almost the same but the distance of the zone from point P and the value of θ increases as we move from lower to higher n. The amplitudes u_1, u_2, u_3 etc. of 1st, 2nd, 3rd etc. zones at point P will be, therefore, in gradually decreasing order as shown in figure 7.9. The opposite directions of alternate amplitudes correspond to the phase change of π between consecutive zones.

Thus the resultant amplitude at P can be written as

$$u_p = u_1 - u_2 + u_3 - u_4 + ...+ (-1)^{n+1} u_n$$

The positive and negative signs on the right hand side between alternate terms of this equation may be ascribed to the fact that the disturbances produced by two consecutive zones at P will be out of phase by π radians.

As the disturbances at P due to various zones are of gradually decreasing magnitudes, the amplitude due to any zone may be taken approximately equal to the average of the amplitudes due to the preceding zone and the succeeding zone. That is, we can take



In equation the last term on right hand side will be positive if n is odd and negative if it is even. We can rewrite equation as

$$u_p = \frac{u_1}{2} + \left(\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right) + \left(\frac{u_3}{2} - u_4 + \frac{u_5}{2}\right) + \cdots \ldots \ldots \ldots \ldots \ldots$$

If n is odd, then $u_p = \frac{u_1}{2} + \left(\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right) + .. + \left(\frac{u_{n-3}}{2} - u_{n-1} + \frac{u_n}{2}\right) + \frac{u_n}{2}$

$$u_p = \frac{u_1}{2} + \frac{u_2}{2}$$

If n is even, then $u_p = \frac{u_1}{2} + \left(\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right) + .. + \left(\frac{u_{n-3}}{2} - u_{n-2} + \frac{u_{n-1}}{2}\right) - u_n$

$$u_p = \frac{u_1}{2} + \frac{u_{n-1}}{2} - u_n$$

If the number of half period zones formed is large enough then due to gradually decreasing amplitudes of zones, the values of u_n and u_{n-1} may be neglected as compared to u_1, and therefore we can write

$$u_p \simeq \frac{u_1}{2}$$

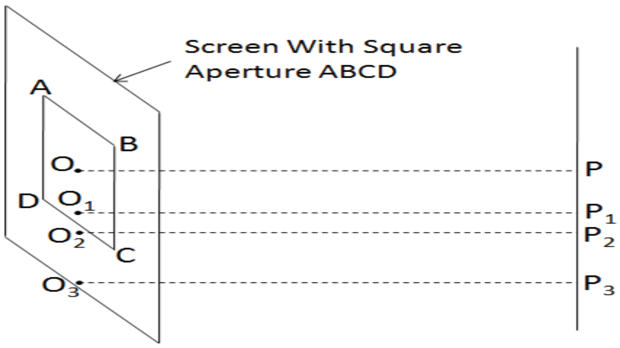
And the intensity at point *P*, therefore, may be given by

$$u_p \simeq \frac{u_1^2}{4}$$

Thus the resultant amplitude produced by whole of the wavefront is equal to one half of that produced by the first zone and the intensity due to the entire wavefront is the one fourth of that by the first zone.

RECTLINEAR PROPAGATION OF LIGHT

With the help of the theory discussed so far we can explain the rectilinear propagation of light. Suppose a plane wavefront of monochromatic light is made to incident on a screen with square aperture *ABCD* and whole of the wavefront except *ABCD* portion is blocked by the screen as shown in the figure. Let *P* be a point at which the intensity of the light is required and its pole *O* with respect to the aperture *ABCD* is well inside from the edges. Taking *O* as centre if we draw the half period zones in the incident wavefront then the number of the wavefronts will be quite large before they intersect the edges *AB*, *BC*, *CD*, and *DA*. Thus practically all the effective zones are exposed and the resultant amplitude at *P* due to aperture *ABCD* is given by equation. This amplitude is equal to the one half that due to the first zone and since the areas of these zones are extremely small, we can consider the light to be travelling along a straight line along *OP*. This condition is the same as if the screen with square aperture *ABCD* was removed.

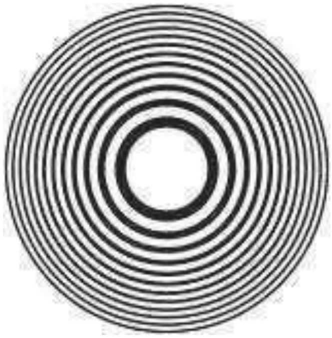


The poles O_1 and O_2 of the points like P_1 and P_2 on the screen lie very close to edges of the aperture *ABCD*. If we draw the half period zones around these poles then some of the zones are obstructed and some are exposed. Thus, there will be neither uniform illumination nor complete darkness at points P_1 and P_2. For the points near the edges the light, therefore, enters into the geometrical shadow region. The point P_3 is well inside the geometrical shadow region and its pole is O_3. Since the amplitude at a point due to a zone decreases on increasing its order, almost all the effective zones around O_3 are cut off. The amplitude reaching at P_3 is nearly zero and there is a complete darkness. This is possible only when light travels along a straight line.

From the above-mentioned facts this may be concluded that there is almost uniform illumination at the points whose poles lie well inside the edges of the aperture and complete darkness at the points whose poles lie well outside the edges. This strongly supports the rectilinear propagation of light. There is a slight deviation from the rectilinear path for the points whose poles lie very close to the edges. However, due to very small value of the wavelength of light this region is very small as compared to whole of the aperture. Thus, as a whole the propagation of the light may be considered along a rectilinear path.

ZONE PLATE

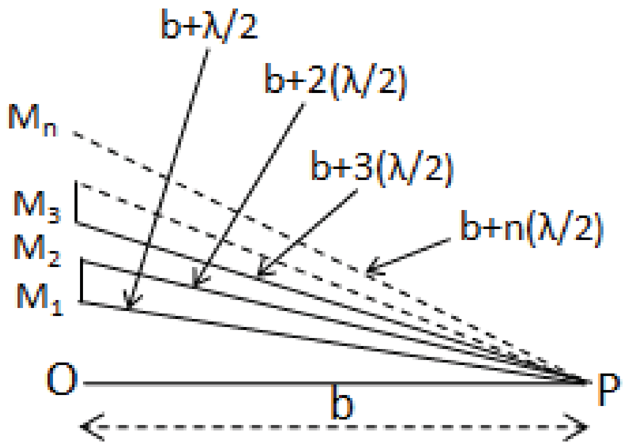
A zone plate is a device used to focus light; however, zone plates use diffraction instead of refraction or reflection as in case of lenses and curved mirrors. It is a specially designed diffraction screen consisting of a large number of half period zones. In the honor of Augustin-Jean Fresnel they are sometimes called Fresnel zone plates. It is constructed in such a way that every alternate zone blocks the light incident on it. In other words, we can say that it consists of alternate opaque and transparent set of radially symmetric rings (zones).



The zones can be spaced so that the diffracted light constructively interferes at the desired focus. The light may be cut off either by even numbered zones or by odd numbered zones. When the light is obstructed by even numbered zones the plate is known as positive zone plate and when obstructed by odd numbered zones it is called negative zone plate. These two kinds of zone plates are shown in figures.

Construction and Theory of Zone Plate

From equation, it is evident that the radii of half period zones are proportional to square roots of natural numbers. Thus, to construct a zone plate, we draw the concentric circles of the radii proportional to square roots of natural numbers on a white paper. The alternate regions between the circles are painted black. If the odd numbered zones are painted black then drawings appear like figure and if even numbered zones are covered with black ink then the drawing looks like figure. Suppose the drawing resembles with figure. If we take a reduced photograph of it then the developed negative resembles with figure. This negative is then used as a zone plate.



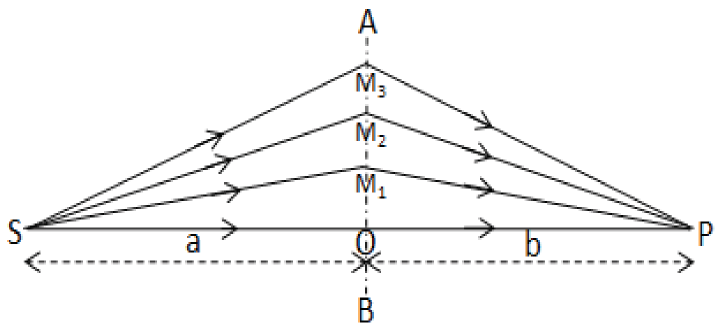
If a beam of light is made to incident on such a zone plate normally and a screen is placed on the other side of this plate to get an image then the maximum brightness is obtained at a particular point of the screen. Suppose this point is *P* at a distance of *b* units from the zone plate as shown in figure.

Only upper half portion of the zone plate is shown in this figure. If *ß* is the wavelength of light used then radius of the first zone ($OM_1=r_1$), second zone ($OM_2=r_2$) etc are given by $r_1 = \sqrt{b\lambda}$ and $r_2 = \sqrt{2b\lambda}$ etc.

The general expression for radius may be written as

$$b = \frac{r_n^2}{n\lambda}$$

Since the wavelength of light has a small value, the sizes of the zones are usually very small as compared to the distance of the light source from the zone plate. Hence OM_1, OM_2, OM_3 etc are extremely small as compared to distance a (source S to zone plate AB separation). But to make the points M_1, M_2, M_3 etc distinct and to show the complete figure the distances are not taken in this ratio in figure. Because of this reason the incident wavefront may be taken as a plane wavefront.



Now suppose even numbered zones are opaque to incident light then from equation, the resultant amplitude reaching at P may be written as (n is odd)

$$u_p = u_1 + u_3 + u_5 + \ldots\ldots\ldots + u_n$$

In this case if all the zones are transparent to light then from equation, the resultant amplitude at P is given by

$$u_p = u_1 - u_2 + u_3 - u_4 + \ldots\ldots\ldots + u_n$$

For large value of n, from equation, we have,

$$u_p = \frac{u_1}{2}$$

If we compare the values of the resultant amplitudes from equations, we find that, when the even numbered zones are opaque the intensity at point P is much greater than that when all the zones are transparent to incident light. Again, from the above discussion we can state that a zone plate behaves like a converging lens. The focal length of the zone plate may be given by

$$f_n = b = \frac{r_n^2}{n\lambda}$$

Therefore, the focal length of a zone plate varies with the wavelength of incident light that is why it is called a multi foci zone plate. For this reason, if white light is made to incident on a zone plate different colors come to focus on screen at different points and it shows chromatic aberration.

Action of a Zone Plate

AB is the section of zone plate perpendicular to the plane of paper, S is the point light source at a distance a from zone plate and point P is on the screen placed at a distance b from the zone plate. As compared to the radii of zones, the distance of source from the zone plate is extremely large and therefore we can take approximation as $SO \approx SM_1 \approx SM_2 \ldots\ldots = a$. The position of the screen is chosen such that the light rays reaching at P from successive zones have a path difference of $\lambda/2$. We can write

$$SO + OP = a + b$$

$$SM_1 + M_1P \approx SO + (OP + \lambda/2) = a + b + \lambda/2$$

Similarly, $$SM_2 + M_2P = a + b + 2\,\lambda/2$$

Now from right angle triangle ΔSOM_1, we have,

$$SM_1^2 = (SO^2) + (M_1O^2)$$

Since $a >> r_1$, expanding above and neglecting higher order terms, we get,

$$SM_1 = a\left(1 + \frac{r_1^2}{2a^2}\right) = a\left(1 + \frac{r_1^2}{2a}\right)$$

Proceeding in a similar way we can obtain,

$$M_1P = \left(b + \frac{r_1^2}{2b^2}\right)$$

Substituting values of SM_1 and M_1P from equations in the left hand side of equation, we get,

$$\left(a + \frac{r_1^2}{2a}\right) + \left(b + \frac{r_1^2}{2b}\right) = a + b + \frac{\lambda}{2}$$

$$r_1^2\left(\frac{1}{a} + \frac{1}{b}\right) = \lambda$$

$$r_2^2\left(\frac{1}{a}+\frac{1}{b}\right)=2\lambda$$

Proceeding similarly for higher order zones, we obtain

$$r_n^2\left(\frac{1}{a}+\frac{1}{b}\right)=n\lambda$$

Now comparing the zone plate with converging device like convex lens and using similar sign convention for the distances of the object and image from the lens, the equation may be modified as

$$\left(\frac{1}{b}-\frac{1}{a}\right)=\frac{n\lambda}{r_n^2}$$

This equation is similar to the lens equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Thus a zone plate behaves like a $f_n=\frac{r_n^2}{n\lambda}$ converging lens of focal length Thus the focal length of zone plate depends on the number of zones and the wavelength of light used.

Multiple Foci of Zone Plate

A zone plate has a multiple foci. In order to prove this, taking an object at infinity, i.e. at $a=\infty$ in equation, we $r_n^2 = bn\lambda$ and therefore, the area of n^{th} zone is given by

$$A_n = \pi r_n^2-\pi r_{n-1}^2 = \pi[n\lambda b-(n-1)\lambda b]=\pi\lambda b$$

Since the object is at infinity, the light rays will be parallel to principal axis and the image will be formed at the principal focus at a distance $b=\frac{r_n^2}{n\lambda}$ from the zone plate.

If we take a point P_3 at a distance $b/3$ from the zone plate somewhere in between O and P then the area of each half period zone with respect to P_3 will now becomes $\pi\lambda(b/3)$, that is, one third to the previous case. Thus each zone, in this case, can be assumed to contain three half period elements corresponding to P_3. If the amplitude due to these elements are represented by m_1, m_2, m_3 etc. then the first zone (amplitude u_1) will consist of the first three elements (amplitudes m_1, m_2 and m_3), second zone (amplitude u_2) will consist of the next three elements (amplitudes m_4, m_5 and m_6) etc. Again similar to half period zones there will be a phase difference of π between the successive elements. Thus while adding the amplitudes; the m_1 will be taken positive, m_2 as negative etc. Substituting the values of u_1, u_2, u_3 etc. with m_1, m_2, m_3 etc., equation changes to

$$u_{p_3} = (m_1 - m_2 + m_3) + (m_7 - m_8 + m_9) + (m_{13} - m_{14} + m_{15}) + \dots\dots\dots$$

$$= \frac{1}{2}(m_1 + m_3 + m_7 + m_9 + m_{13} + m_{15} + \dots\dots)$$

Here it should be noted that each of the amplitudes m_1, m_2, m_3 etc is one third of u_1, u_2, u_3 etc. If we compare the equations, we find that the intensity reaching at P_3 is sufficiently large but is less than that reaching at P. Thus the image of S is also formed at P_3 and therefore, it may be taken as the second focal point. The second focal length is given by

$$f_3 = \frac{r_n^2}{3n\lambda}$$

Similarly the images of S can be formed on points P_5, P_7, P_9 etc. but with decreasing intensity. The distance of these points from the zone plate are $\frac{r_n^2}{5n\lambda}, \frac{r_n^2}{7n\lambda}, \frac{r_n^2}{9n\lambda}$.

Zone plate has multiple foci point is

$$f_1 = \frac{r_n^2}{n\lambda}, f_3 = \frac{r_n^2}{3n\lambda}, f_5 = \frac{r_n^2}{5n\lambda} \quad etc.,$$

Comparison of Zone Plate and Lens

Some of the features of zone plate are similar to a lens and in some it has dissimilarity. The following are the resemblance and differences between the two.

(i) Similar to a lens, a zone plate forms an image of an object placed on its axis. The same sign convention is used while representing the distance of the object and image in both the cases.

(i) The focal length formula in terms of distance of object and image for zone plate is $\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{1}{f}$ and for the convex lens is $\left(\frac{1}{v} - \frac{1}{u}\right) = \frac{1}{f}$ which are identical.

(ii) The image due to a convex lens is more intense as compared to that due to a zone plate.

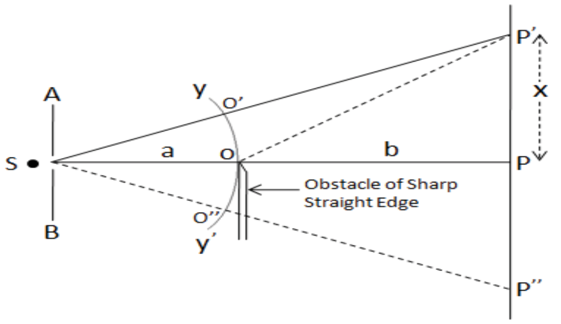
(iii) The convex lens has a focal length given by $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ which depends on wavelength (refractive index varies with wavelength) and the focal length of zone plate f $= \frac{r_n^2}{n\lambda}$ also varies with wavelength.

(iv) A convex lens has one focal length for a fixed wavelength while a zone plate has a number of foci at which the images of diminishing intensities are formed.

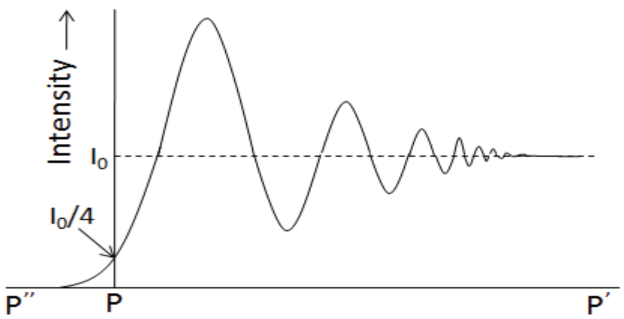
DIFFRACTION AT A STRAIGHT EDGE

To show the diffraction effect of a straight edge, the light from a monochromatic light source *S* is passed through a narrow slit *AB* and a sharp edge of an opaque obstacle like blade is placed in its path as shown in figure. The slit, opaque obstacle and screen $P'P''$ are parallel to each other and perpendicular to the plane of the paper. The sharp edge is placed in such a way that the line joining the slit to edge *O* when reproduced meet the screen at *P* and *OP* is normal to screen.

In the absence of diffraction of light due to sharp straight edge there should be a uniform illumination above point *P* and complete darkness below it. As we move towards *P'*, unequally spaced bright and dark bands are obtained near *P*. On further moving towards *P'*,i.e. with increasing value of *x* the intensity reaches a steady value I_o resulting a uniform illumination. Because of the diffraction effect, the light enters to a certain distance below *P* (towards *P''*) in the geometrical shadow region.



In this region the intensity of light decreases to zero very rapidly without forming maxima and minima in a small but finite distance as shown in intensity distribution curve of figure. If the average intensity is I_o then at point *P* on the screen (corresponding to the edge) it reduces to $I_o/4$. This all is due to the diffraction of light produced by sharp straight edge.



Theoretical Analysis

Suppose we want to find the resultant at any point, say *P'*, on the screen. The pole of the wavefront *YY'* with respect to point *P'* will be *O'*. With *P'* as centre if we draw the circles of radii $O'P'+\lambda/2$, $O'P'+2\lambda/2$, $O'P'+3\lambda/2$ etc, the wavefront is divided into half period strips. Thus for point *P*', the wavefront is divided in two similar parts; one above point *O'* another below it. The light from entire upper half portion of the wavefront reaches to *P'*. The resultant due to this will be equivalent to one half to that due to first half period strip, i.e. $m_1/2$. Now the number of half period strips within the lower half portion of the wavefront, i.e. *O'O* will depend on the position of the point *P'* on the screen. Suppose the lower half portion contains only one half period strip then the amplitude due to it at *P'* will be only m_1 and therefore, the total amplitude at *P'* by whole of the exposed wavefront is given by $\frac{m_1}{2}+m_1$

If *O'O* contains two, three, four etc half period strips then the resultant amplitude at *p'* is given by $\frac{m_1}{2}+m_1 - m_2$,$\frac{m_1}{2}+m_1 - m_2 + m_3$,$\frac{m_1}{2}+m_1 - m_2 + m_3 - m_4$ position of *P'* gives the position of first minimum, position of second maximum and the position of second minimum respectively. Thus at point *P'*, a maximum or a minimum is formed according as *O'O* contains odd or even number of half period strips.

As we move away from *P* towards *P'* alternate maxima and minima are obtained. From the previous discussion we see that the amplitude or intensity of these maxima and minima are comparable, hence the bands have a poor contrast. If the point of consideration is at a sufficiently large distance from *P* then entire upper half and a large number of half period strips of the lower half are exposed. The diffraction bands merge together to produce uniform illumination. The resultant amplitude at the point of consideration, in this case, is therefore,

$$\frac{m_1}{2} + \frac{m_2}{2} = m_1$$

Positions of Maximum and Minimum Intensities

The path difference between the rays $O'P'$ and OP' is given by

Δ = OP'-O'P' = $(OP^2 + PP'^2)^{1/2}$-SP' -SO'

= $(b^2 + x^2)^{1/2} - \ [(a+b)^2 + x^2 - a]$

Y*Y'* is the spherical wavefront of the point light source *S* with *S* as a centre, thus *SO'* = *SO* = a, is the radius of the sphere.

In actual experimental set up we have, $x<<b$. Thus taking b out (common) from the first term and $(a+b)$ out from the second term on the right hand side of the above equation, expanding the series and neglecting higher order terms, we obtain

$$\Delta= (2n-1)\frac{\lambda}{2}$$

For minima we have,

$$\Delta= 2n\frac{\lambda}{2}$$

Comparing the equations, we get

$$x_n = K\sqrt{2n-1}$$

we have, $x_1 = K$, $x_2 = K\sqrt{3}$, $x_3 = K\sqrt{5}$ etc. Thus the separations between successive maxima are $x_2 - x_1 = 0.732K$, $x_3 - x_2 = 0.504K$, $x_4 - x_3 = 0.409$ etc. We see that with increasing order of maxima the separation between consecutive maxima decreases and the fringes come closer. The same is true for minima.

Intensities at Various Positions

(i) Intensity at the Edge of Geometrical Shadow

In figure the edge of geometrical shadow is represented by P. The pole of this edge at wavefront is point O, which is nothing but the edge of sharp obstacle. Thus with respect to the edge of geometrical shadow region (point P), the incident wavefront can be divided in two parts; one above point O (OY) and other below point O (OY'). The light from the entire upper half portion of the wavefront reaches to point P while the light from the lower half portion of the wavefront is completely cut off by sharp edge obstacle. The resultant amplitude at P, in this case, is $m_P = m_1 - m_2 + m_3 + m_4 - \ldots\ldots\ldots$, *which is* $m_1/2$. Thus the resultant intensity at P is $m_1^2/4 = I_o/4$. Where I_o is the value of intensity at P in the absence of obstacle.

(ii) Intensity at a Point Inside the Geometrical Shadow

If the point of consideration is inside the geometrical shadow region, then the pole of the point will be below point O, i.e. in the wavefront region OY'. Suppose we take a point P'' then its pole will be O''. In this case the complete lower half portion and most of the upper half portion of the wavefront is obstructed by the obstacle. Only a small part of the upper half portion of the wavefront (OY) is exposed. As we move down gradually from point P inside geometrical shadow, the first, the first two, the first three etc. half period strips of the upper half of the wavefront are obstructed and the amplitudes are thus $m_2/2$, $m_3/2$, $m_4/2$ etc. respectively. The intensities, therefore, will be $(m_2/2)^2$, $(m_3/2)^2$, $(m_4/2)^2$ etc. respectively.

Since the amplitudes m_1, m_2, m_3 etc. are in decreasing order of magnitude, the intensity of light decreases rapidly as we move inside the geometrical shadow. This is because of the fact that most of the effective half period strips of the upper half portion of wavefront are cut off.

When the two objects are very near to each other or they are at very large distance from our eye, the eye may not be able to see them as separate. If we want to see them separate, optical instruments such as telescope, microscope etc. (for close objects) and prism and grating etc. (for spectral lines) are employed. Even if we assume that the instruments employed are completely free from all optical defects, the image of a point object or line is not simply a point or line but it is a diffraction pattern with a bright central maximum and other secondary maximum, having minima in between of rapidly decreasing intensity. Thus, an optical instrument is said to be able to resolve two-point objects if the corresponding diffraction patterns are distinguishable from each other.

The ability of an optical instrument to resolve (i.e. view separately) the images of two close point source is known as resolving power.

Plane Transmission Grating

A diffraction grating is made by making many parallel scratches on the surface of a flat piece of some transparent material. It is possible to put some large number of scratches per cm on the material. For example, the grating to be used has 6,000 lines per cm on it. The scratches are opaque but the areas between the scratches can transmit the light through. Thus, a diffraction grating becomes a multitude for the source with parallel slit, when light falls upon it.

The formula for diffraction grating:

Consider two rays that emerge making the angle θ with the straight through the line. Constructive interference will occur if the difference in their two path lengths is an integral multiple of their wavelength λ i.e.,

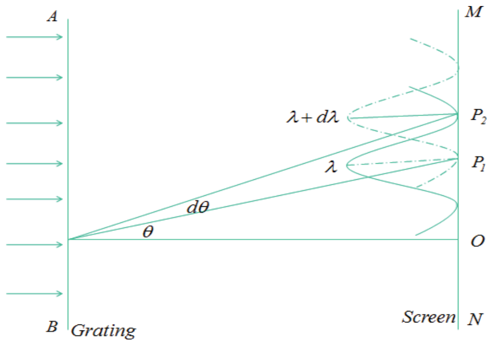
$$\text{Now, } n\,\lambda = d\sin(\theta)$$

$$\text{where } n = 1, 2, 3, \ldots$$

This is known as the **Diffraction Grating Equation**.

In this formula, θ is the angle of emergence at which a wavelength will be bright. Also, d is the distance between slits. Obviously, d = 1/N, where N is the grating constant, and it is the number of lines per unit length. Also, n is the order of grating, which is a positive integer, representing the repetition of the spectrum.

Furthermore, a complete spectrum could be observed for n = 1 and another complete spectrum for n = 2, etc., but at the larger angles.



Theory

If a parallel beam of monochromatic light is incident normally on the face of a plane transmission diffraction grating, bright diffraction maxima are observed on the other side of the grating. These diffraction maxima satisfy the grating condition :

$$(a + b)\times\sin\theta_n = n\lambda \text{ , …(1)}$$

where (a+b) = the grating element (=2.54/N, N being the number of rulings per inch of the grating), θn = the angle of diffraction of the n^{th} maximum n = the order of spectrum which can take values 0, ±1, ±2, ±3 ……. λ = the wavelength of the incident light Clearly, the diffraction is symmetrical about θ0 = 0. If the incident beam contains different colours of light, there will be different θn corresponding to different λ in the same order n. By measuring θn and knowing N, λ can be calculated.

Chromatic resolving power of a grating is defined as its power of distinguishing two nearby spectral lines and is defined as Chromatic R.P = $\frac{\lambda}{\Delta\lambda}$

Where Δλ is the separation of two wavelengths which the grating can resolve; the smaller the value of Δλ, the larger the resolving power. Employing Rayleigh's criterion for the limit of resolution, one can show in the case of a grating.

The angular dispersion or dispersive power of a grating is defined as the rate of change of angle of diffraction with the change of wavelength in a particular order of the spectrum. Differentiating eqn. (1) with respect to λ, we get λ

$$\frac{d\theta}{d\lambda} = \frac{n}{a + bcos\theta}$$

shows that for a given small wavelength difference Δλ the angular separation Δθ is directly proportional to the order n. When θ is small (less than 60°), cosθ is constant and hence Δθ is proportional to Δλ. Such a spectrum is called a normal spectrum.

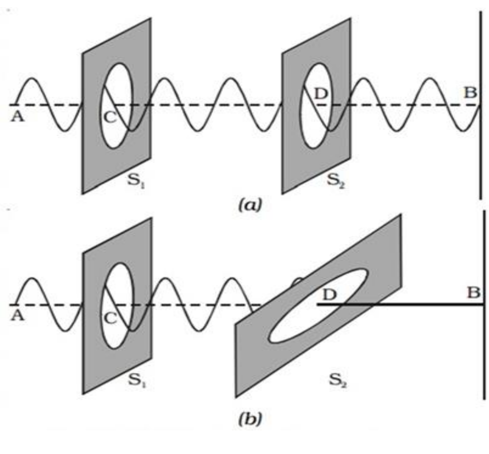
UNIT – IV POLARISATION

The phenomenon of interference and diffraction has proved the wave nature of light but it doesn't tell us regarding the character (whether longitudinal or transverse?) and nature of vibration of light (whether linear, circular, elliptical or torsional). It is the phenomenon of polarization that shows that light wave is definitely transverse in nature. Light wave is a transverse electromagnetic wave made up of mutually perpendicular, fluctuating electric and magnetic fields vibrating perpendicular to each other as well as direction of propagation too.

In general, natural light is unpolarized in nature, i.e., it consists of a very large number of wavelength with electric vector vibrates in all possible planes with equal probability when ordinary light is allowed to pass through tourmaline crystal, the vibration of electric field are confined only to one direction in a plane perpendicular to the direction of propagation of light. This light which has acquired the property of one sidedness and whose electric field vibration lacks in symmetry called polarized light. This polarization of light means departure from complete symmetry about the direction of propagation.

Two Slit Analogy of Polarized Light

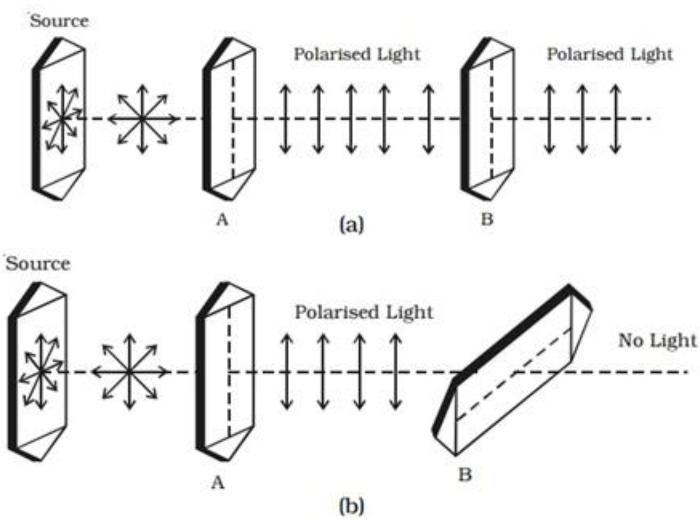
Let S_1 and S_2 be the two-slit adjacent to each other (figure 10.1). A string AB passing through the slit and attached to a fixed point at B. Now if we move the end A of the string up and down perpendicular to AB, the string vibrates and a transverse wave propagates along CD. If slit S_2 is placed parallel to S_1, vibration passing through S_1 and S_2 reach the end B without any change in amplitude (figure 10.1(a)). Now let us rotate slit S_2 as S_2 becomes perpendicular to S_1, the vibration pass through the slit S_1 undisturbed as before but are not able to pass through S_2 and therefore the string does not vibrate between S_2 and B.



In the intermediate positions of the slit S_2, the vibration are partially transmitted and partly stopped, reaching the end B with diminished amplitude. The variation of amplitude as the rotating slit S_2 is only because of transverse vibration in the string. On the other hand the end A is moved to and fro parallel to the length of the string instead of up and down setting longitudinal vibration, we see that the rotation of any of the slide about AB as axis does not affect the passage of vibrations and hence vibration reach at B with undiminished amplitude. Therefore, we can say that variation in amplitude of vibration passing through S_2 on rotation of S_2 signifies the transverse vibration of the string.

Now if we replace the slit with tourmaline crystal and string with a source of light, exactly similar phenomenon is observed Fig. When light from source S falls in a tourmaline crystal A cut parallel to its crystal axis the emergent light is slightly coloured. Now if we place a similar cut crystal B in the path of beam partially to the axis of crystal A we observed that emerging light is still coloured and the intensity is maximum. If now keeping crystal ‘A' fixed we rotate the crystal B about the axis the intensity decreases and becomes zero when B is perpendicular to A. By further rotation light reappears and becomes maximum again when ‘A' and ‘B' again becomes parallel.

This variation in amplitude proves that light is transverse in nature. Whereas, if it is longitudinal, there shouldn't be any variation by rotating the crystals discussed in two slit analogy. It also shows that light vibration after passing through rotating slit are not symmetrical about the direction of propagation.



Comparison of Unpolarized Light and Polarized Light

Unpolarised Light	Polarised Light
Unpolarized light consists of waves with planes of vibration equally distributed in all directions about the direction of propagation.	Polarized wave consists of waves with light vector vibrating in a single plane perpendicular to the direction of propagation.
Unpolarized light is symmetrical about the ray direction.	Polarized light is asymmetrical about the ray direction.
Unpolarized light is produced by conventional light sources	Polarized light is generally obtained from unpolarized light with the help of polarizer.

Double Refraction

Splitting of a beam of unpolarised light into two refracted beam is called double refraction or birefringence. This phenomenon was first discovered by a Dutch philosopher E. Bartholinius in the year 1669.

Explanation: As shown in figure, if an image of ink dot on a sheet of paper is viewed through a calcite crystal we observe two images instead of usual one and if crystal is rotated slowly one of the two images rotates around the other stationary image.

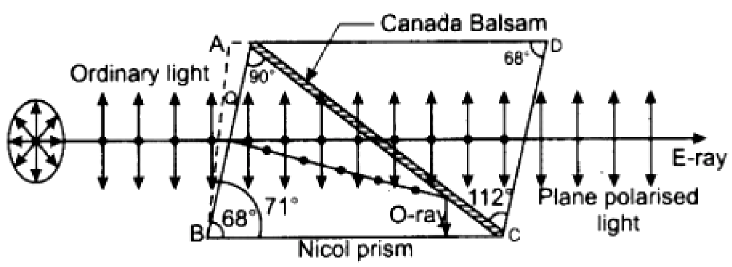


image as ordinary image. The refracted ray which produces the rotating image is called extraordinary ray (E-ray) and the image as extraordinary image. O-ray obeys the ordinary laws of refraction while E-ray doesn't.

Now let us explain this. As shown in Figure., if a narrow beam of light AB is incident on calcite crystal with angle of incidence *i*, it is split up into two rays (instead of one as usual) inside the crystal. The two images are obtained on the screen corresponding to O- ray and E-ray. O-ray travels along the side BC with angle of refraction r_1 while the E- ray travels along BD make angle of refraction r_2 and both the rays emerges out along CO and DE which are parallel to each other and also to incident beam.

$$\text{By snell's Law } \mu = \frac{sin\ i}{\sin r} = \frac{velocity\ of\ light\ in\ Vaccum}{Velocity\ of\ light\ in\ \ medium}$$

Therefore, refractive index for O- ray and E- ray are respectively

$$\mu_0 = \frac{sin\ i}{\sin r_1} \text{ and } \mu_e = \frac{sin\ i}{\sin r_2} \text{ in case of crystal } r_1 < r_2$$

Hence $v_o < v_e$ i.e inside the calcite like crystal the velocity of E-ray is greater than O-ray or we can say that Inside the calcite crystal E-ray travels faster in comparison to O-ray.

It is also observed that for O-ray the refractive index is constant for all angle of incidence as it obeys ordinary law of refraction while for E-ray the refractive index is not constant but varies with angle of incidence. Therefore it can be easily concluded by this observation that the O-ray travels with the same speed in all the directions within the crystal while the E-ray travels with the different speeds in the different directions. Also both E-ray and O-ray shows variations in intensity when passes through rotating tourmaline crystal prove that both ordinary and extraordinary ray obtained by double refraction are plane polarized. It is also observed that their plane of polarization are at right angles to each other with O-ray vibrations perpendicular to the plane of paper while vibrations of E-ray is parallel to the plane of paper.

NICOL PRISM

Nicol prism is an optical device fabricated from calcite crystal for producing and analyzing plane polarized light named after its inventor William Nicol who designed it in 1820. Its action is based on phenomenon of double refraction. It's constructed in such a way that O-ray is eliminated by total internal reflection and we get only the plane polarized E-ray coming out of the Nicol.

Construction

A rhomb of calcite crystal with length AB three times as long as its breadth CD is obtained by cleavage from the original crystal. The ends faces are grounded until they make an angle of 68^0 instead of 71^0 in natural crystal. Then the crystal is cut into two parts along a plane perpendicular to the principal axis as well as to the new end surfaces AB and CD (figure 11.4). Thus the two parts are grounded and polished optically flat than again cemented together with a layer of Canada balsam. Canada balsam is a clear transparent material whose refractive index (μ_{cb} = 1.55) lies between the refractive index for calcite for the O-ray (μ_o = 1.66) and E-ray (μ_e = 1.486) for sodium yellow light of mean wavelength λ = 5893 Å.

Action of Nicol Prism

When unpolarised light is incident on the prism parallel to AB. it suffers double refraction and splits into O-ray and E-ray as shown in figure. When these ray strikes at Canada balsam layer (which is denser than calcite for E-ray and less dense for O-ray) O-ray travels from an optically denser to a rarer medium it is totally reflected in case the angle of incidence is greater than a certain critical value (critical angle). This reflected ray is completely absorbed as the tube containing the crystal is coated black. E–ray is not reflected as it travels from an optically rarer to a denser medium hence E-ray is transmitted through the prism. Thus Nicol prism acts as a polarizer.

Now the reason behind natural angle 71^0 is reduced to 68^0 and choosing length three times to its width enables the O-ray to fall at the Canada balsam layer at an angle greater the critical angle.

Let θ is the critical angle for the O-ray. Than refractive index of O-ray with respect to Canada balsam layer is

$$\frac{\mu_o}{\mu_{cb}} = \frac{1.68}{1.55}$$

The condition for total internal reflection is

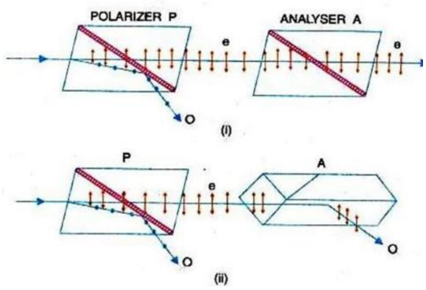
$$\sin\theta = \frac{1}{\mu} = 0.935 = 69°$$

Nicol Prism as Polarizer and Analyzer

When two nicol prisms P and A are arranged coaxially adjacent to each other. First Nicol P acts as a polarizer and other acts as analyzer. Such a combination of polarizer and analyzer is called polariscope.

If unpolarized ray of light is incident on the first Nicol P, E-ray is transmitted with its vibration directly lying in the principal section of P. The state of the polarization of the light can be analyzed by another Nicol called analyzer. If principal section of analyzer A is parallel to principal section of polarizer P, E-ray is transmitted through analyzer A without any hindrance. In the case of Parallel Nicols the intensity of emergent E-ray is maximum (figure). If the analyzer A is gradually rotated the intensity of E-ray decreases in accordance with Malus law.

In case the angle between principal sections of two Nicol is 90^0 they are called crossed Nicol. In the case E-ray when it enters the analyzer acts as O-ray inside the prism hence totally reflected at balsam layer hence no light comes out of the analyzer A (figure 11.5(b)). If A is further rotated the intensity of emergent light from A will go on increasing and is becomes maximum when its principal section is again parallel to that of P.



In this way the Nicol prism A is used to analyze the plane polarized light produced by Nicol prism P hence named Analyzer and polarizer respectively.

Limitation of Nicol Prism

A Nicol prism cannot be used for highly convergent and divergent beams. It is found that the angle of incidence is limited for14^0 above which the O ray is also transmitted. It is also found that E- ray also totally reflected if the angle is greater than 14^0. Thus to avoid the transmission of O-ray and total internal reflection of E-ray the angle between the extreme rays of the incoming beam is limited to 2x14 = 28^0.

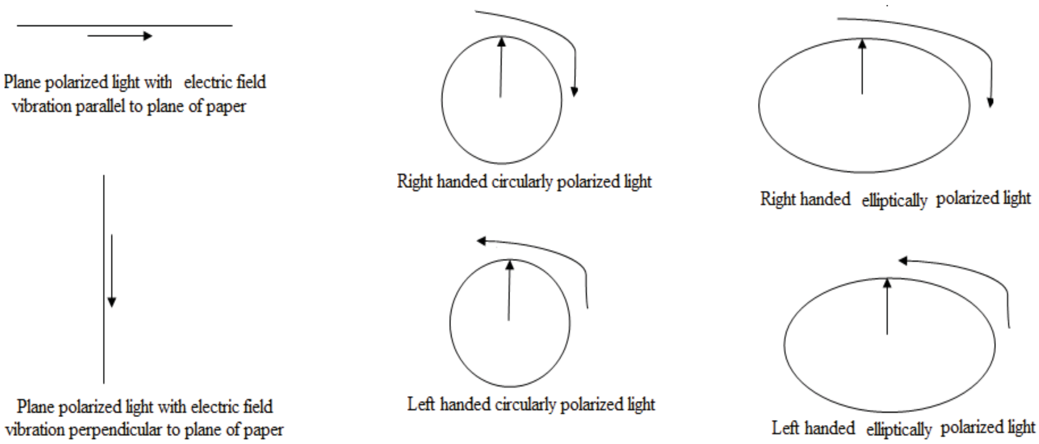
TYPES OF POLARIZATION

The polarization of a light wave describes the shape and locus of the tip of the E vector at a given point in space as a function of time. Depending upon the locus of the tip of the E vector light may be exhibit three different states of polarization. They are

1. Plane polarized light
2. Circularly polarized light
3. Elliptically polarized light

CONCEPT OF PLANE POLARIZED LIGHT, CIRCULARLY POLARIZED LIGHT AND ELLIPTICALLY POLARIZED LIGHT

As we know that light is an electromagnetic wave consist of mutually perpendicular electric and magnetic field vector both are vibrating perpendicular to the direction of propagation of light wave. Also electric vector is dominating and is responsible for optical effects of wave hence the electric vector is also called light vector.



(a) Plane or linearly polarized light (b) Circuarly polarized light (c) elliptically polarized light

As mentioned earlier, unpolarized light have vibrations along all possible straight lines perpendicular to the direction propagation the light. Light which has acquired property of one sidedness is called polarized light. Therefore plane polarized light is not symmetrical about the direction of propagation but the vibrations of light vector (electric vector) are confined to a single direction i.e. along a line of course perpendicular to direction of propagation also known as linearly polarized light. We can also say that in plane or linearly polarized light the magnitude of light vector changes but its orientation remains unchanged. Usually light is a mixture of plane polarized light and unpolarized light, known as partially plane polarized.

On the other hand a light wave is circularly polarized if the magnitude of light vector remains constant but its orientation rotates at a constant rate about the direction of propagation so that the tip of the light vector traces a circle. It completes one evolution within one wavelength. Circularly polarized wave may be considered as the result of superposition of two mutually perpendicular plane polarized waves having equal amplitude but a phase difference of 90^0. If rotation of tip of light vector E is seen clockwise it is called right circularly polarized light if it rotates anticlockwise the wave is said to be left circularly polarized light.

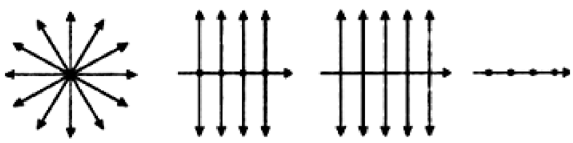
Similarly a light wave is called elliptically polarized if the magnitude of light vector as well as its orientation changes about the direction of propagation so that the tip of the light vector traces an ellipse. Elliptically polarized wave may be considered as the result of superposition of two mutually perpendicular plane polarized waves having different amplitude and not in same phase.

Like circularly polarized light if rotation of tip of light vector E is seen clockwise it is called right elliptically polarized light if it rotates anticlockwise the wave is said to be left elliptically polarized light.

PICTORIAL REPRESENTATION OF PLANE POLARIZED LIGHT

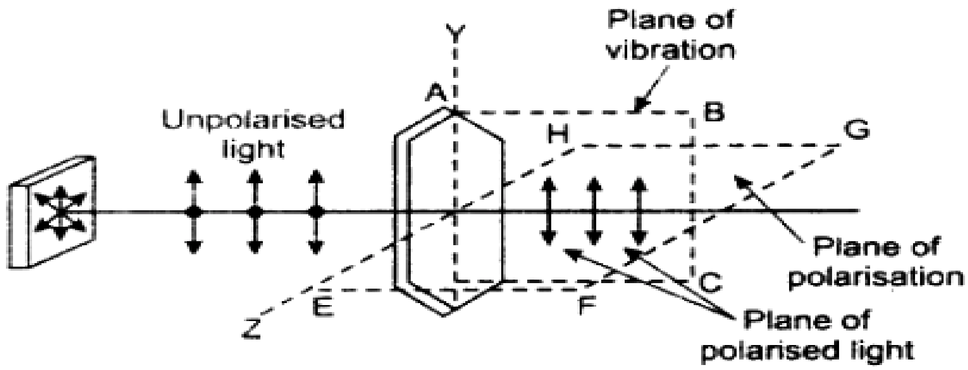
Ordinary or unpolarized light obtained from any source consists of vibration of electric field vector in al possible plane perpendicular to the beam direction i.e. the electric field vibrations are symmetrical. Unpolarized light can be considered as consisting of two sets of vibrations- one set vibrating in one plane and other perpendicular to it. It may also be represented respectively by arrows and dots. Hence unpolarized lights pictorially represented end view would be as shown in Figure.

In a plane polarized light the vibrations of electric vector are along a single straight line thus having departure of complete symmetry. When electric field vector or light vector of plane polarized light has vibration in the plane of the paper they are represented by arrows as shown in Figure 10.4 (b).When the vibrations of light vector are in a direction perpendicular to the plane of the paper they are represented by dots as shown in Figure.



PLANE OF VIBRATION AND PLANE OF POLARIZATION OF PLANE POLARIZED LIGHT

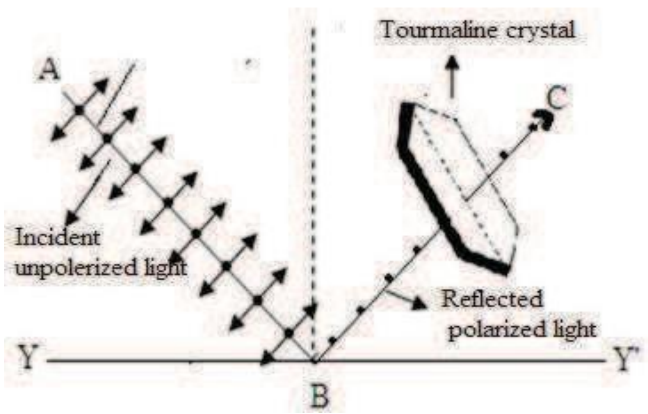
As discussed, earlier plane polarized light may be defined as the light in which the electric vector or light vector vibrates along a fixed straight line in a plane perpendicular to the direction of propagation. However, to define the properties of plane polarized light completely we have to define two planes, one containing the vibrations and other perpendicular to it, as the properties of plane polarized light differ with respect to these two planes. The plane containing the direction of vibration and direction of propagation or the plane in which the vibration takes place is called the plane of vibrations and a plane perpendicular to plane of vibration is called plane of polarization. We can also define plane of polarization as the plane passing through the direction of propagation and containing no vibrations.



As shown in Figure the plane ABCD is the plane of vibration and the plane EFGH is the plane of polarization.

PLANE POLARIZED LIGHT BY REFLECTION

Producing plane polarized light by reflection is the simplest way. In 1808 E.L. Malus noticed that when natural or unpolarized light is incident on a transparent medium like glass or water the reflected light is partially plane polarized. The degree of polarization depends upon the incident angle on the surface and upon the material of the surface. At a certain angle of the incidence depending upon the nature of the reflecting surface the reflected light is completely plane polarized. This angle of incidence is called angle of polarization or polarizing angle.



Here it must be noted that light reflected from the metallic surface contained a variety of vibration directions; i.e. reflected light from metallic surface is unpolarized. But if light is reflected from dielectric surface such as glass, water etc. is linearly polarized. If the extent of linear polarization is large enough as glare from field of snow on bright sunny day, its glare from the surface may be almost blinding to human eye.

When light wave is incident on a boundary between two dielectric materials, part of it is reflected and part of it is transmitted. Let a beam of unpolarized light incident along AB on a glass surface YY' and reflected as BC. To show that reflected light is plane polarized a tourmaline crystal is placed in the path of reflected ray BC and rotated about BC as axis. It is observed that the transmitted light shows variation in intensity. It proves partial plane polarized nature of reflected ray. At polarizing angle of incidence, reflected ray from crystal is almost completely extinguished shows that maximum percentage of plane polarized light. Further rotation about BC as axis, the intensity of reflected beam through crystal is twice maximum and minimum in one complete rotation depending upon whether axis of crystal is perpendicular or parallel to plane of incidence respectively. It indicates that the light vibration in reflected beam is perpendicular to plane of incidence.

However, this particular method of polarizing light is not very advantageous as only a small portion of incident beam is reflected therefore the intensity of the reflected beam is very small.

A retardation plate, also known as plate retarders or wave plates, is an optically transparent birefringent crystal which resolves a beam of unpolarized light into two orthogonal components (ordinary light rays and extra ordinary light rays); change the relative phase difference between the components; then recombines the components into a single beam with new polarization characteristics. These plates are very useful to produce different kind of polarized light and convert one type of polarized light to other.

A retardation plate is generally a plane-parallel plate of a birefringent crystal like quartz, mica, magnesium fluoride and sapphire, with the optic axis in the plane of the surface. It is oriented so that incident polarized light may be resolved into components projected along the optic axis and perpendicular to it. These two components will experience a relative phase shift (retardation) proportional to the thickness of the plate. When the fractional part of this retardation is a nonzero value, the waveplate modifies the polarization state of incident polarized light polarizations from one state to another. A waveplate does not polarize light, but modifies the state of polarized light. Further, a relative phase shift produced by the retardation plate is subject to the availability of both components (E- rays and O- rays) of incident light which are parallel and perpendicular to its optic axis.

The amount of retardation can be expressed as birefringence times thickness. Birefringence ($\mu_o - \mu_e$) varies with temperature and wavelength. For a given temperature and wavelength, one can form the retardation plate or the wave plate for producing a wanted phase shift between the two components (ordinary light rays and extra ordinary light rays) for specific purposes. The retardation plates are mainly of two types as follows.

Quarter Wave Plate

A retardation plate of such a thickness that it produces a path difference of λ/4 (quarter of the wavelength of incident light) or a phase difference π/2 between the two components (ordinary light rays and extra ordinary light rays) of incident light beam passing through it, is known as quarter wave plate. Hence, for a quarter wave plate

$$\text{Path difference} = (\mu_o \sim \mu_e)\ x\ \text{thickness}$$

or $$ß/4 = (\mu_o \sim \mu_e)\ t\ ,$$

Where, *t* is the thickness of the doubly refracting crystal or birefringent crystal

$$t = \frac{\lambda}{4(\mu_0 - \mu_e)}$$

Half Wave Plates

If the thickness of the retardation plate is such that it produces a path difference of λ/2 (half of the wavelength of incident light) or a phase difference π between the two components (E-ray and O-ray) of incident light beam passing through it, then this retardation plate is known as half wave plate. Hence, for a half wave plate

$$\text{Path difference} = (\mu_o \sim \mu_e)\ x\ \text{thickness}$$

$$ß/2 = (\mu_o \sim \mu_e)\ t\ ,$$

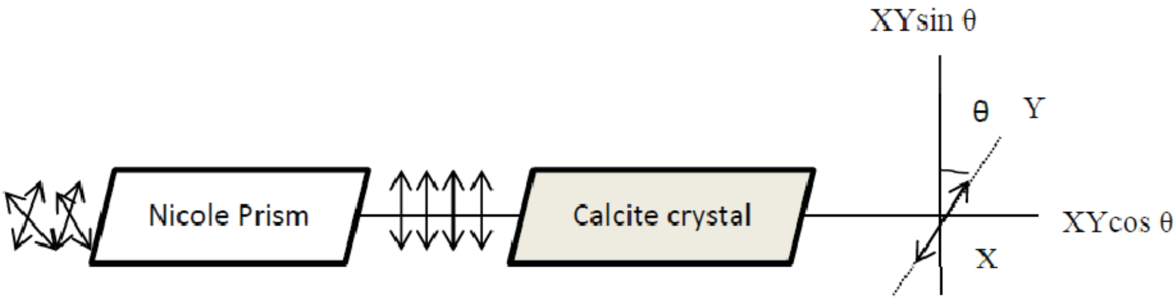
Where *t* is the thickness of the doubly refracting crystal or birefringent crystal

$$t = \frac{\lambda}{2(\mu_0 - \mu_e)}$$

APPLICATION OF RETARDATION PLATES: THEORY OF THE PRODUCTION OF POLARIZED LIGHT

You have well understood so far about a birefringent crystal and its property of birefringence (production of E-rays and O-rays with their plane of vibrations perpendicular to each other), when an unpolarized light passes through it. Now, consider a beam of plane polarized light having their vibrations of electric field along one axis only perpendicular to the direction of propagation of beam, falls normally on a birefringent crystal (like calcite crystal) cut with optic axis parallel to its faces. Let $A = XY$ be the maximum amplitude of incident light which makes angle θ with optic axis of crystal. The plane polarized light split up in two components ordinary light (O-ray) and extra ordinary light (E-ray) with amplitude $XY \cos\theta$ along XA and $XY \sin\theta$ along XB.

As discussed earlier, these two rays travel with different speeds but in same direction inside the crystal.



Due to this, the E-ray and O-ray develops a path difference or phase difference (δ) after emerging from the crystal. If A is the amplitude of the incident wave and plane polarized wave (XY=A in figure), the E-rays and O–rays vibrates along perpendicular directions with amplitude $A\cos\theta$ and $A\sin\theta$ with a phase difference of δ depending upon the thickness of the crystal. The form of resultant vibration in the resultant vibration will be given by the resultant of these two components having simple harmonic vibrations in perpendicular directions with angular frequency ω and phase difference δ.

Let us consider the equations of these two components as

$$X = A\cos\theta \sin(\omega t + ð) \ldots\ldots(1)$$

$$Y = A\sin\theta \sin\omega t \ldots\ldots(2)$$

Let $A\cos\theta = a$ and $A\sin\theta = b$, the above equation reduces to new form

$$X = a \sin(\omega t + ð) \ldots\ldots(3)$$

$$Y = b\sin\omega t \ldots\ldots(4)$$

From equation, we have, $\sin\omega t = Y/b$ and $\cos\omega t = \sqrt{1 - \frac{y^2}{b^2}}$

$$\frac{x}{a} = \sin\omega t \cos\delta + \cos\omega t \sin\delta$$

$$\frac{x}{a} - \frac{y}{b}\ cos\delta = \sqrt{1 - \frac{y^2}{b^2}} sin\delta$$

Squaring on both sides, simplifying we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\left(\frac{XY}{ab}\right) cos\delta = \ sin^2\delta$$

Thus the resultant wave equation is a general equation of ellipse. The polarization state of emerging light beam will depend on δ. We will discuss the special cases of above equation depending upon the phase difference (δ) between two components after emergence from the crystal:

Production of Plane Polarized Light

Case 1: When $\delta = 0, 2\pi, 4\pi, 6\pi$ .... , we have, $\sin \delta = 0$ and $\cos \delta=1$

From equation, we have,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\left(\frac{XY}{ab}\right) = 0$$

$$\left(\frac{\mathrm{x}}{\mathrm{a}} - \frac{\mathrm{y}}{\mathrm{b}}\right)^2 = 0$$

$$y = \frac{b}{a}X$$

This is an equation for straight line with slope. Hence, the resultant light beam will be plane polarized with plane of vibration in a direction $\tan^{-1}(b/a)$. If the amplitudes of both components are same ($a = b$), the resultant polarized light will vibrate at an angle of 45^0 with x axis.

Case 2: When $\delta = \pi, 3\pi, 5\pi, 7\pi$ .... then $\sin \delta = 0$ and $\cos \delta= - 1$

From equation, we have,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\left(\frac{XY}{ab}\right) = 0$$

$$\left(\frac{\mathrm{x}}{\mathrm{a}} + \frac{\mathrm{y}}{\mathrm{b}}\right)^2 = 0$$

$$\left(\frac{\mathrm{x}}{\mathrm{a}} + \frac{\mathrm{y}}{\mathrm{b}}\right) = 0$$

Hence, the resultant light beam will be plane polarized with plane of vibration in a direction $tan^{-1}\left(\frac{b}{a}\right)$ If the amplitudes of both components are same ($a = b$), the resultant polarized light will vibrate at an angle of 135^0 with x axis.

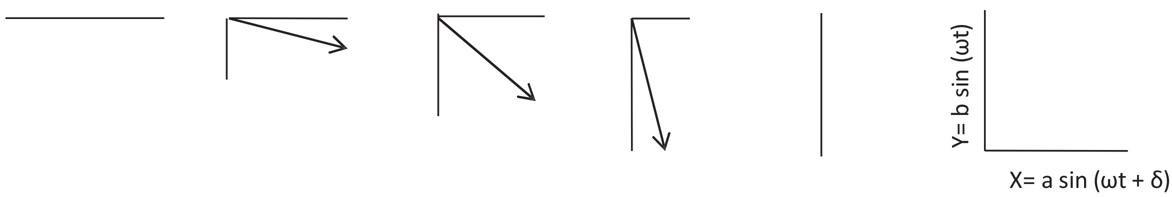
Production of Elliptical and Circular Polarized Light

Case1: When δ = 3π/2, 7π/2, 11π/2, 15π/2........... , we have, sin δ = -1 and cos δ=0.

From equation, we get,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

This is an equation of symmetrical ellipse with major and minor axes as *a* and b. Hence, the resultant light beam will be elliptical polarized. From figure 12.2, we draw some position of the both of components (E-ray and O-ray) having phase differences of δ = 3π/2, 7π/2, 11π/2, 15π/2....... . When the displacement of one component (X= *a* sin (ωt + δ)) is maximum, the other has zero displacement (Y= b sin ωt = 0). As the displacement of the first component comes below its maximum displacement, an increase in the displacement of second component starts with phase differences of δ = 3π/2, 7π/2, 11π/2, 15π/2. This process continues up to when the displacement of X- component becomes zero and the Y- component shows maximum displacement. The resultant of these intermediate displacements of both the component shows a clockwise or right handed rotation. Hence, it can be concluded that the resultant wave will be right handed elliptically polarized light.



If the amplitudes of both the components (E–ray and O–ray) are same, i.e., *a* = b. The above equation of ellipse reduces to following form:

$$X^2 + Y^2 = a^2$$

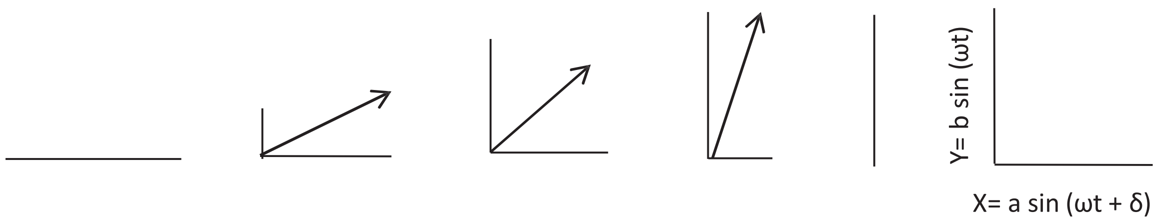
This is an equation of circle. The emergent light will be right handed (clockwise) circularly polarized light if the of E-ray and O-ray has same amplitudes and maintains a phase difference of δ = 3π/2, 7π/2, 11π/2, 15π/2 . . .

Case2: When δ = π/2, 5π/2, 9π/2, 13π/2........... , we have, sin δ = -1 and cos δ=0

From equation, we obtain,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

This is again an equation of symmetrical ellipse with major and minor axes are *a* and b, but the resultant is a combination of E- rays and O- rays with a phase differences which are different to Case 1. In this case, the resultant light beam will be left-handed elliptical polarized. From figure 12.2, we can understand this situation. When the displacement of one component ($X= a \sin(\omega t + \delta)$) is maximum, the other has zero displacement ($Y= b \sin(\omega t) =0$). As the displacement of the first component comes below its maximum displacement, an increase in the displacement of second component will appears with a phase difference of $\pi/2, 5\pi/2, 9\pi/2, 13\pi/2$...... .



This process continues to the condition where the displacement of X-component becomes zero and the Y-axes has maximum displacement. The resultant of these intermediate displacements of both the component shows an anticlockwise or left handed rotation. Hence, it can be concluded that the resultant wave will be right handed elliptically polarized light. If the amplitudes of both the components (E– ray and O– ray) are the same, i.e., $a = b$ then above equation of ellipse reduces to following form

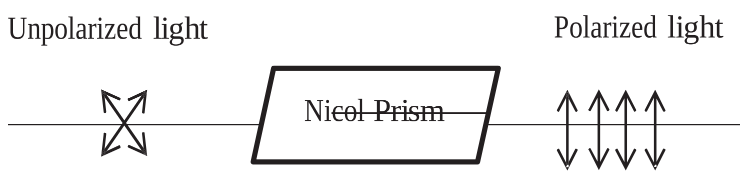
$$X^2 + Y^2 = a^2$$

This is an equation of circle. The emergent light will be left handed (anti clockwise) circularly polarized light if the of E- ray and O- ray has same amplitudes and maintains a phase difference of $\delta = \pi/2, 5\pi/2, 9\pi/2, 13\pi/2$..... .

EXPERIMENTAL ARRANGEMENTS FOR THE PRODUCTION OF POLARIZED LIGHT

Plane Polarized Light

When a beam of unpolarized monochromatic light is allowed to pass through a polarizer (Nicol Prism), it splits up into two components (O-rays and E-rays). Both rays are plane polarized with perpendicular plane of vibrations. The O-rays, follows the ordinary laws of refraction, total internally reflected and get absorbed inside the Nicol prism. The only ray coming out from the Nicol prism is E-rays, which is plane polarized. The Experimental arrangement to produce plane polarized light contains a Nicol prism which is placed after the unpolarized source light.



Circularly Polarized Light

Circularly polarized light is the resultant of two mutually perpendicular vibrations of equal amplitudes with a phase difference of π/2. The unpolarized light is allowed to fall on a Nicol prism. The emergent light from Nicol is plane polarized light. Another Nicol prism (Analyzer) is placed at some distances from polarizing Nicol with crossed position. In this position no light is transmitted from second Nicole (Analyzer), hence the field of view appears dark. A quarter wave plate (QWP) is then introduced between two Nicole in the path of plane polarized light such that polarized beam falls normally on the rotating quarter wave plate. The amplitude of polarized beam on entering the quarter wave plate is splits up into two mutually perpendicular components and receives a phase difference of π/2 after emergence. Now, the quarter wave plate rotated and the position of the quarter wave plate is such that the vibration in the incident polarized light make an angle of 45^0 with the optic axis of the plate. In this position the plane polarized light on entering the quarter wave plate is split up into two components (E-ray and O-ray) of equal amplitudes.

In this way the resultant of two perpendicular components with equal amplitudes having a phase difference of π/2 produces a circularly polarized light after emerging out from the quarter wave plate.

Elliptically Polarized Light

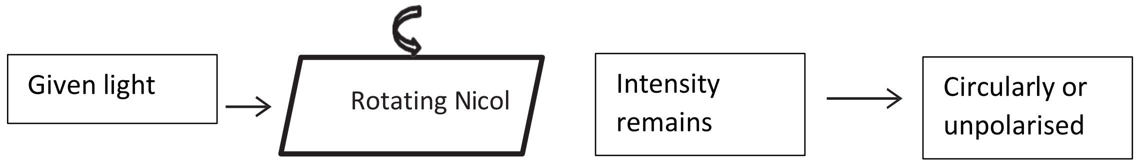
This is a more general case than circular polarization, in which there is a phase difference of π/2 between the two components. Elliptical polarization is the result when the components are equal with non-quarter-wave phase difference.

To obtain the elliptically polarized light, the experimental arrangement and undergoing processes is the same as shown in figure 12.5. The only change is the rotation of quarter wave plate should maintain such that the vibrations of light incident on it makes any angle other than 45^0. This makes two perpendicular components with unequal amplitudes having a phase difference of π/2 and the resultant wave produces an elliptically polarized light.

APPLICATION OF RETARDATION PLATES: ANALYSIS OF POLARIZED LIGHT

Analysis of Plane Polarized Light

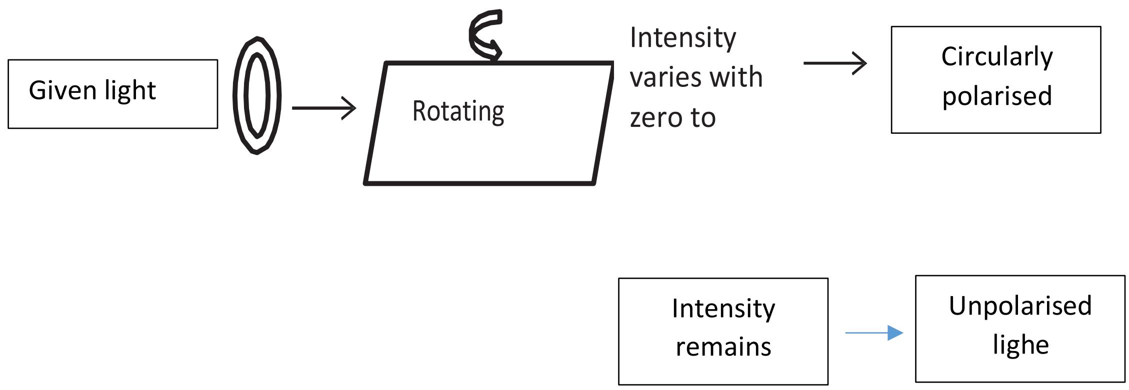
When a given light is allowed to pass through a rotating Nicol prism and the intensity of emergent light remains unchanged, this shows that incident light beam is either circularly polarized or un-polarized.



Now, given light is first passed through a quarter wave plate (QWP). If the given light is circularly polarized light, then it splits into two components (E-ray and O-ray) and because of QWP there is a phase difference of $\pi/2$ developed between them in addition to existing phase difference of $\pi/2$ which exists because of it being a circularly polarized light.

Therefore, the total phase difference between the E-rays and O-rays becomes 0 or π, which results in rectilinear vibrations and the emergent light is now plane polarized. If we pass this light again through a rotating Nicol prism, the intensity of emergent light will vary with zero to maximum intensity.

If the intensity of emergent light will remain unchanged even after passing a quarter wave plate and rotating Nicole, this shows that the incident light beam is unpolarised light.

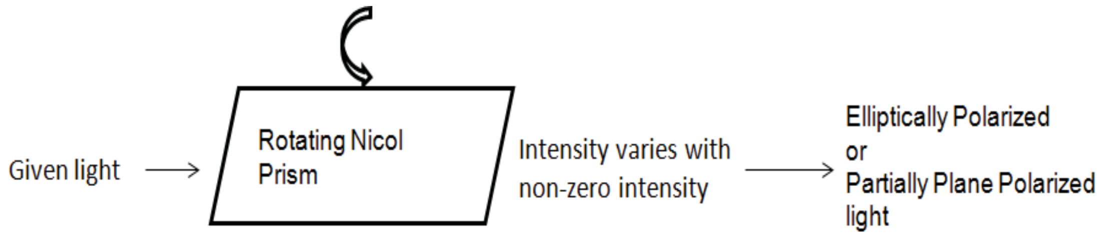


If the intensity of emergent light will remain unchanged even after passing a quarter wave plate and rotating Nicole, this shows that the incident light beam is unpolarised light.

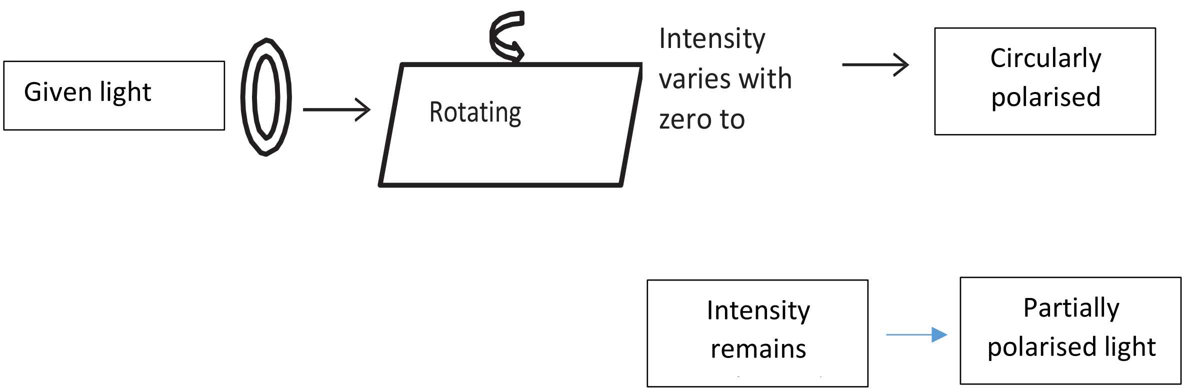
Analysis of Elliptically Polarized Light

When beam of incident light is passed through the rotating Nicol prism and the intensity of emergent light varies from maximum to non-zero intensity, the incident light may be elliptically polarized or partially plane polarized light.

Now, given light is first passed through a quarter wave plate (QWP). If the given light is elliptically polarized light, then it splits into two components (E-ray and O-ray) and because of QWP there is a phase difference of $\pi/2$ developed between them in addition to existing phase difference of $\pi/2$ which exists because of it being an elliptically polarized light. Therefore, the total phase difference between the E- rays and O-rays becomes 0 or π, which results in rectilinear vibrations and the emergent light is now plane polarized.



If we pass this light again through a rotating Nicol prism, the intensity of emergent light will vary with zero to maximum intensity. On the other hand, if incident light is a partially plane polarized light, a mixture of plane polarized and unpolarized light, then after passing through a quarter wave plate it cannot become plane polarized and thus the intensity through the Nicol prism will change from maximum to non-zero.



ROTATORY POLARIZATION

Rotatory polarization or optical activity is an ability of any substances to rotate the plane of incident linearly polarized light. The substances rotate the plane of vibration in clockwise direction are known as dextrorotatory substances and those substances produce a counterclockwise rotation are known as levorotatory substances. This property was discovered in quartz in 1811 by Arago. Two different crystalline structures of quartz produce d-rotatory and l-rotatory behavior. In the case of many naturally occurring organic compounds such as turpentine, sugar and, tartaric acid optical activity also shows rotatory polarization in the liquid state. This shows that the activity is associated with the individual molecules themselves.

FRESNEL'S THEORY OF OPTICAL ROTATION

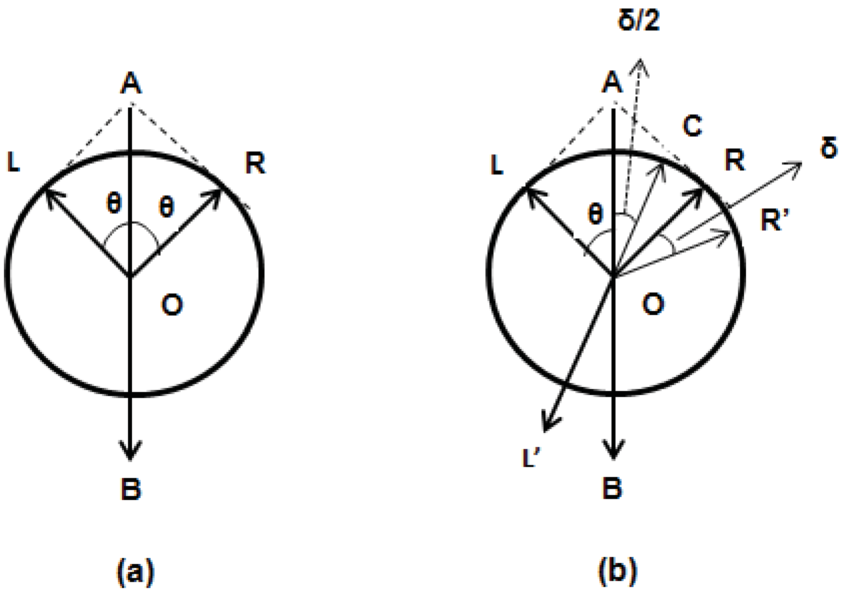
Fresnel's theory of optical rotation is based on the fact that a linearly plane polarized light consists of resultant of two circularly polarized vibrations rotating in opposite directions with same angular velocities. Fresnel's made the following assumptions

(1) When a beam of plane polarized light enters in a crystal along the optic axis, it is broken up into two circularly polarized vibrations, one right handed and the other left handed.

(2) When a plane polarized light enters a crystal of an optically inactive substance (like calcite) along optical axis, it breaks up into two circularly polarized vibrations rotating in opposite direction with same angular frequency or velocity such that resultant of these two vibrations at all point of time is along the optic axis. The Vibrations of clockwise direction rotation in (in figure 13.2 (a)), are represented by OR and vibration rotating in counterclockwise direction are represented by OL. The resultant OR and OL at all the point of time will be along AB.

(3) In case of an optically active substance (like Quartz), a linearly polarized light on entering the crystal is resolved into two circularly polarized vibrations rotating in opposite direction with different angular velocity or frequency. In case of left-handed optically active quartz crystal, anticlockwise vibrations travel faster, while in case of right handed optically active, quartz crystal, clockwise vibrations travel faster.

Consider a right-handed quartz crystal in which clockwise component travels faster than left-handed component. Suppose at any instant of time, right-handed component traverse angle δ greater than left-handed component as shown in figure 13.2 (b). The new position of resultant of L and OR will be along CD i.e., plane of vibration of light has been rotated through angle $\delta/2$ towards right after passing through quartz crystal. The angle $\delta/2$ depends upon thickness of crystal.



(a) For Optically Inactive Crystals: when linearly plane polarized light enters a calcite crystal it gets resolved into two circularly polarized vibrations. One is moving anticlockwise with same angular frequency or velocity. As each circularly polarized vibration further consist of two rectangular components having phase differences π/2, so for clockwise circular vibration

$$x_1 = a\, sin\theta = a\, sin\, \omega t$$

$$y_1 = a\, cos\, \theta = a\, cos\omega t$$

For anticlockwise circular vibration

$$x_2 = -a\, sin\, \theta = -a\, sin\, \omega t$$

$$y_2 = a\, cos\, \theta = a\, cos\, \omega t$$

From above, the resultant displacement of vibrations along x-axis and y-axis respectively are given as,

$$x = x_1 + x_2 = a\, sin\, \theta - a\, sin\, \theta = 0$$

$$y = y_1 + y_2 = a\, cos\, \omega t + a\, cos\, \omega t = 2a\, cos\, \omega t$$

Hence resultant vibration has amplitude 2a and is plane y-axis, i.e., along original direction AB. Hence two oppositely circularly polarized vibrations give rise to a plane polarized vibration.

(b) For Optically Active Crystal (Quartz): When linearly plane polarized light enters quartz crystal (right-handed), it gets resolved into circularly polarized vibrations moving in opposite direction with different angular frequency or velocity. In case of right-handed crystal

(c) clockwise vibrations travel faster than anticlockwise vibrations. Let at any instant of time anticlockwise vibrations has traversed angle e and clockwise vibrations has traversed angle(θ + ð). Therefore, component of two circular vibrations at that instant of time will be, for clockwise vibration

$$x_1 = a\, sin\, (\omega t + ð)$$

$$y_1 = a\, cos\, (\omega t + ð)$$

For anticlockwise circular vibration

$$x_2 = -\, a\, sin\, \omega t$$

$$y_2 = a\, cos\, \omega t$$

From resultant displacement of vibrations along x-axis and y-axis respectively are given as

$$X = x_1 + x_2 = a\, sin\, (\omega t + ð) - a\, sin\, \omega t = 2a\, sin\, ð/2\, cos\, (wt + ð/2)$$

$$Y = y_1 + y_2 = a\, cos\, (\omega t + ð) + a\, cos \omega t = 2a\, cos\, ð/2\, cos\, (\omega t + ð/2)$$

The resultant vibration along x-axis and y-axis are in same phase, so resultant of these vibrations is plane polarized and makes an angle of δ/2 with original direction AB. Thus, plane of vibrations get rotated through angle ő/2 towards right after passing through a right handed quartz crystal. From above, we get

$$tan\, ð/2 = y/x$$

Angle of Rotation

If μ_R and μ_L be the refractive indices of quartz crystal for right handed and left handed vibrations respectively then optical path difference on passing through a quartz crystal slab of thickness t is given as,

Path difference $=(\mu_L - \mu_R)\, t$

If ß be the wavelength of light used, then phase difference,

$$ð = 2\pi/\lambda\, (\mu_L - \mu_R)\, t.$$

Angle of rotation

$$\theta = ð/2 = \pi/\lambda\, (\mu_L - \mu_R)\, t \text{.................................}(13.2)$$

If v_L and v_R be the velocities of the left- h a n d e d and right- h a n d e d circular vibrations respectively, then, $\mu_L = c/v_L$ and $\mu_R = c/v_R$ where c is the velocity of light.

$$\theta = \frac{\pi t}{\lambda}\left(\frac{c}{v_L} \sim \frac{c}{v_R}\right)$$

If T be the time period, then $c = ß/T$ *or* $1/T = c/ß$

$$\theta = \frac{\pi t}{\lambda}\left(\frac{1}{v_L} - \frac{1}{v_R}\right)$$

In case of left-handed substances (case of dextrorotatory substance), $v_R > v_L$

$$\theta = \frac{\pi t}{T}\left(\frac{1}{v_L} - \frac{1}{v_R}\right)$$

In case of left-handed substances (case of laevorotatory substance), $v_L > v_R$

$$\theta = \frac{\pi t}{T}\left(\frac{1}{v_R} - \frac{1}{v_L}\right)$$

In case of non-optically active substances, $\theta = 0^0$. Hence the direction of vibrations remains unchanged. Such substance does not exhibit the phenomenon of optical rotation.

Experimental Verification

Fresnel's verified his hypothesis by arranging a number of right-handed and left-handed prism of quartz to form a parallelepiped. Axes of the prism are parallel to the bases. When a plane polarized light is incident normally on one face, the beam splits into two circularly polarized beams which are widely separated. These are analyzed by a rotating Nicol. It was observed that the intensity of emergent beam is constant. This establishes the fact that both beams are circularly polarized as assumed by Fresnel.

SPECIFIC ROTATION

When a linearly plane polarized light is passed through an optically active medium/ substance, the plane of linearly polarized light gets rotated through certain angle either towards left or right. The angle through which plane polarized light get rotated depends upon

1. thickness of the medium

2. density of active substance or concentration of solution

3. wavelength of light and temperature.

Hence, mathematically $\theta \propto L$, $\theta \propto C$, $\theta \propto \lambda$,

and $\theta \propto t$

$$\theta \propto L\, C\, \lambda\, t$$

or $\theta = \mathbf{S}\, L\, C\, \lambda\, t$

or $S = \theta / (L\, C\, \lambda\, t)$

Where S is proportionality constant and is known as specific rotation. θ = rotation in degree, L = length of tube in decimeter, C = concentration of solution in gm/ cc, t = temperature and λ. = wavelength of linearly polarized light

For given wavelength and temperature,

$S = \theta / (LC)$ (when L is in decimeter)

or $S = 10\theta / (L\, C)$ (when L is in centimeter)

Hence, specific rotation is defined as the rotation produced by 1 decimeter long solution of concentration 1 g/cc at given temperature for given wavelength. The rotation produced by optically active/medium/substance can be measured by polarimeter.

POLARIMETER

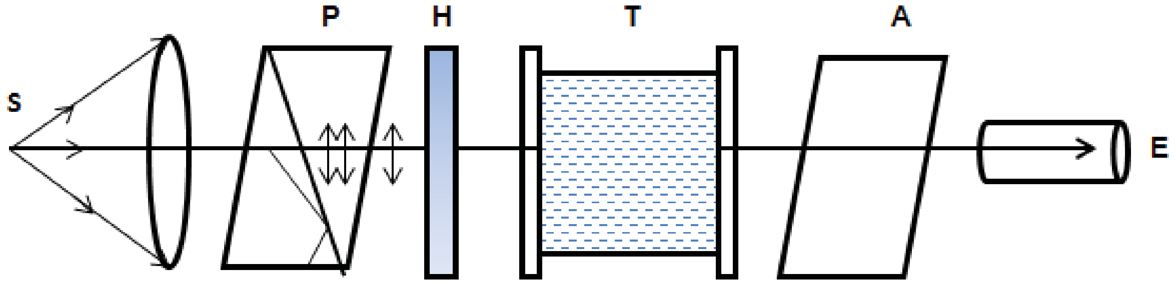
plane polarized light after passing through an optically active substance. When used for finding the optical rotation of sugar it is called a Saccharimeter. If the specific rotation of sugar is known, the concentration of sugar solution can be determined. Generally, there are two types of polarimeters used

1. Laurent's half-shade polarimeter
2. Bi-quartz polarimeter.

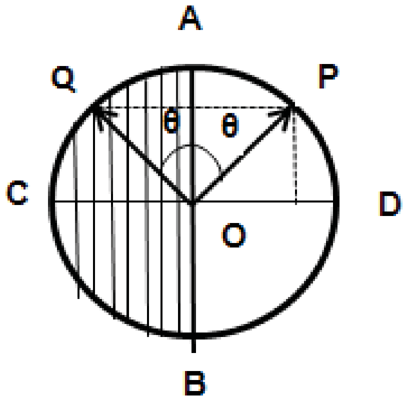
Laurent’s Half-Shade Polarimeter

Construction: The optical parts of a Laurent's half shade polarimeter are shown in figure. A source of monochromatic light (usually a sodium lamp) is placed at the focus of a convex lens, which makes the emerging light rays parallel. This parallel light beam is allowed to fall on a polarizer (usually Nicol prism P) such that the emerging light become plane polarized light. This plane polarized light is then passing through the half shade device H (called Laurent's), a glass tube T containing the solution of optically substance and an analyzer (Nicol prism A). The light is viewed through a telescope E. The analyzing

Nicol A can be rotated about the axis of the tube and its rotation can be measured with the help of a vernier scale on the graduated circular scale C divided in degrees and mounted on telescope.



Working: The Laureate half shade device is a combination of two semicircular plates of glass (ADB) and quartz (ACB) shown in figure 13.4. The quartz plate is cut such that its optic axis becomes parallel to the line of separation AOB. The thickness of the quartz plate is chosen such that it works as half wave plate and offers a phase difference of π between the O- rays and E-rays. The thickness of the glass plate is such that it absorbs the equal amount of light that by the quartz half wave plate.



The source light is first allowed to pass through the polarizer P and emergent plane polarized light with vibrations along OP is incident normally on the half-shade plate (figure 13.4). On passing through the glass half (right half) the vibrations will remain along OP, but on passing through the quartz half (left half), the vibrations along OP split up into two perpendicular components (E -rays and O-rays) along OQ (in left half or through quartz half) and perpendicular to OQ (in right half or glass half). The analysis of the emergent light after Laurent half shade can be done with the help of another Nicol as follows

(1) If the analyzing Nicol is fixed with its principal plane parallel to OP, the plane polarized light through glass half will pass and hence it will appear brighter than the quartz half from which light is partially obstructed.

(2) If the principal plane of the Nicol is parallel to OQ the quartz will appear brighter than the glass half due to the same reason.

(3) When the principal plane of the analyzing Nicol is parallel to AOB, the two halves will appear equally bright. It is because the vibrations emerging out of the two halves are equally inclined to its principal plane and hence two components will have equal intensity.

(4) When the principal plane of the analyzer is at right angle to AOB again the components of OP and OQ are equal. The two halves are again equally illuminated, but as the intensity of the components passing through is small as compared to that in the previous case, the two halves are said to be equally dark.

The eye can easily detect a slight change when the two halves are equally dark. The readings are, therefore, taken for this position.

Application: The Laurent half shade polarimeter can be used to measure the concentration of sugar solution. For this purpose, the following steps to be followed:

(1) Fill the polarimeter tube with water and find the reading on a circular scale corresponding to equally dark position of the half shade device.

(2) Now fill the tube completely with the given sugar solution and again find the reading on the circular scale for equally dark positions of the half shade device.

(3) The difference between the scale readings gives the optical rotation θ produced by the given length l in decimeters of the sugar solution.

(4) If S is the specific rotation of sugar for the same wavelength and at the same temperature, then concentration

$$C = \theta/(LS) \text{ g/cc}$$

Polarimeter is of great importance in the industries for estimating the quantity of sugar in the presence of an optically inactive impurity. A polarimeter calibrated to read directly the percentage of cane-sugar in the solution is called as saccharimeter.

Merits and Demerits of Laurent's Half Shade Polarimeter

- This instrument is suitable for monochromatic light source. It is usually constructed for sodium source.
- When position is adjusted for equally dark halves, it gives fairly accurate observation as slight rotation of the analyzer changes the intensity of two halves.

UNIT V
LASERS

INTRODUCTION

We know current can be amplified by vacuum tube or transistor amplifier. Similarly light waves can also be amplified and is termed as LASER (Light Amplification by Stimulated Emission of Radiation) the fact that there are two kinds of emission, namely spontaneous and stimulated was first predicted by Albert Einstein in 1917. He made this prediction based on the thermodynamic equilibrium between atoms and the radiation field. He further proved that both spontaneous and stimulated emissions are necessary to obtain Planck's quantum radiation law. Charles Towner demonstrated stimulated emission for the first time at microwave frequencies and Theodore Maiman demonstrated it at optical frequencies in a ruby laser in 1960. Within a few months of operation of this device, Javan and his fellow workers constructed the first gas He - Ne Laser. The semiconductor laser was invented in1962. Since then laser action has been obtained in a variety of materials like liquids, ionized gases, dyes, etc.,

Characteristics

(1) Directionality

Ordinary light spreads in all directions and its angular spread is 1m/m. But, it is found that laser is **highly directional** and its angular spread its 1mm/ meter. For example the laser beam can be focused to very long distance with a few divergence (or) angular spread.

Divergence (or) Angular spread is given by $(\phi) = (r_2 - r_1) / (d_2 - d_1)$ degrees

Where d_1 & d_2 are any two distances from the laser source emitted and r_1, r_2 are the radii of the beam spots at a distance d_1 & d_2 respectively

(2) Intensity

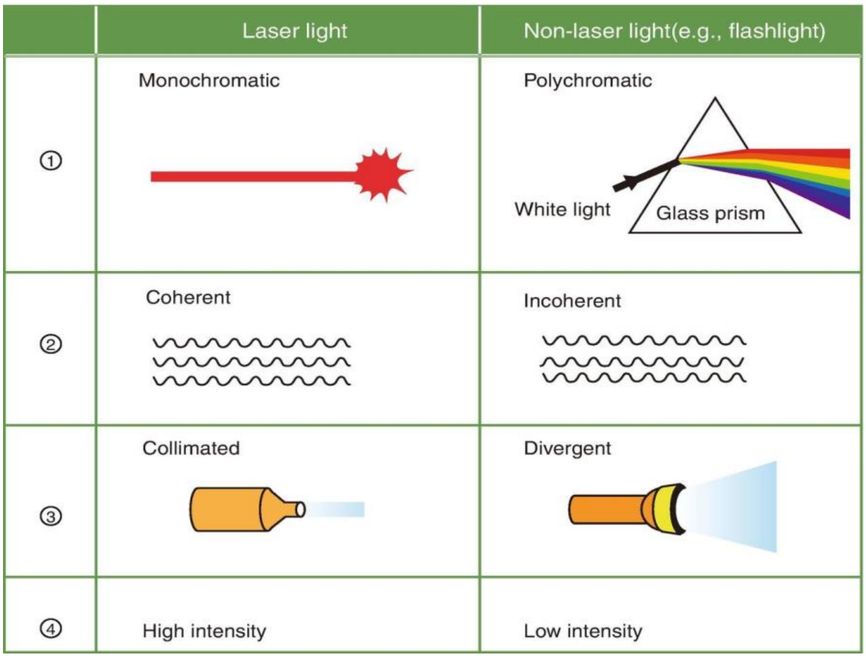
Since an ordinary light spread in all directions, the intensity reaching the target is very less. But in the case of laser, due to high directionality the intensity of laser beam reaching the target is of **high intense beam.** For example, 1mW power of He – Ne laser appears to be brighter than the sunlight

(3) Monochromatic

Laser beam is **highly monochromatic** (i.e.,) the wavelength is single, whereas in ordinary light like mercury vapour lamp, many wavelengths are emitted.

(4) Coherence

In lasers the wave trains of same frequency are in phase (i.e) the radiation given out is in mutual agreement not only in phase but also in the direction of emission and polarization. Thus it is a **Coherent beam.**



Theory of laser

Interaction of light radiation with materials

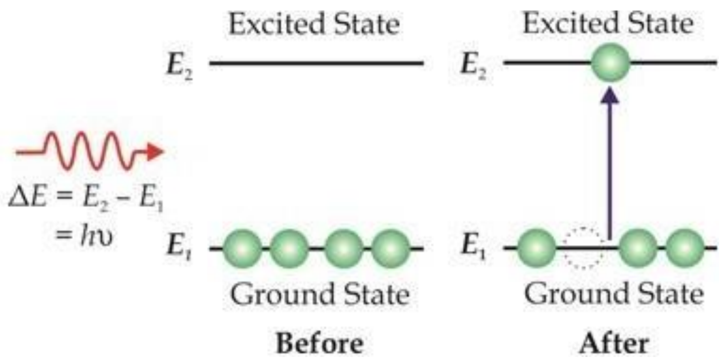
Consider an assembly of atoms in a material which is exposed to light radiation (a stream of photons with energy $h\nu$)

In general, three different processes occur when light radiation interacts with material. They are: (1) Stimulated absorption (2) Spontaneous emission (3) Stimulated emission

1. Stimulated emission

At atom in ground state with energy E_1 absorbs an incident photon of energy $h\nu$ and is excited to higher energy state with energy E_2. This process is called stimulated (or) induced absorption. It occurs only when the incident photon energy $h\nu$ is equal to the energy difference between excited state and ground state

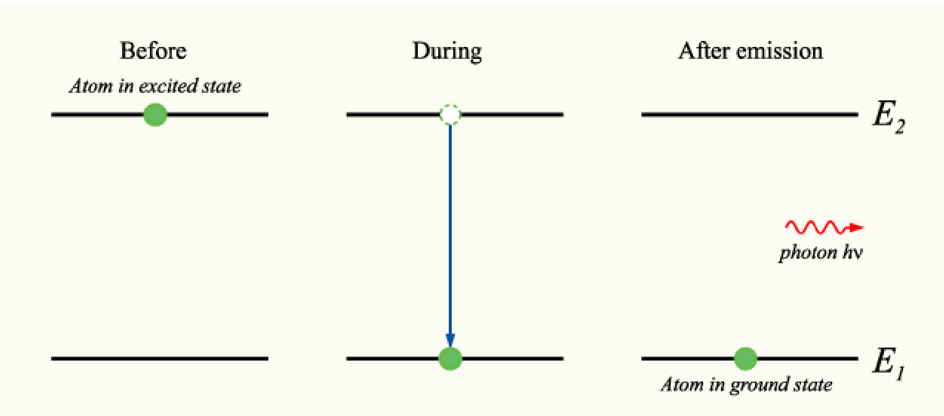
$(E_2 - E_1)$.



For each such a transition, a certain amount of energy hv is absorbed from the incident light beam. The excited atoms do not stay in the higher energy state for a longer time. It is the tendency of atoms in excited state to come to the lower energy state. Thus, the atoms in excited state quickly return to ground state by emitting a photon of energy hv. The emission of photons takes place in two ways : (a) Spontaneous emission (b) Stimulated emission

2. Spontaneous emission

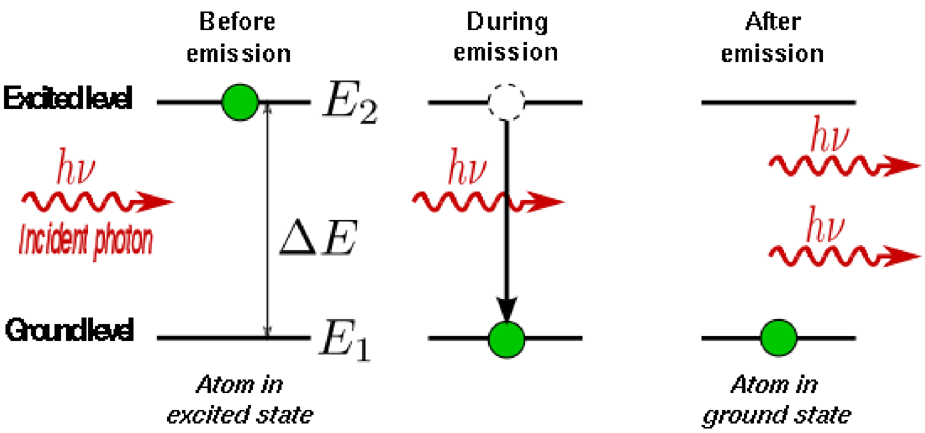
The atom in the excited state E_2 (higher energy state) returns to ground state E_1 (lower energy state) by emitting a photon of energy hv ($\Delta E = E_2 - E_1$) without the influence of any external agency. Such emission of light radiation which is not triggered by any external influence is called spontaneous emission. It is a random and also uncontrollable process.



3. Stimulated emission

Einstein suggested that there must be another mechanism by which an atom in excited state can return to ground state. He found that there is an interaction between the atom in excited state and a photon. During this interaction, the photon triggers the excited atom to make transition to ground state E1. This transition produced a second photon which is similar to triggering photon with respect to frequency, phase and propagation direction.

Such kind of forced emission of photons by the incident photons is called stimulated emission. It is also known as induced emission. It plays a key factor for the working of a laser.



$$E_2 - E_1 = \Delta E = h\nu$$

Principle of Spontaneous & Stimulated emission - Einstein's Quantum theory of radiation

If N_0 is the number of the atoms per unit volume in ground state, then the number of atoms per unit volume in the excited state of energy E is given by Maxwell – Boltzmann distribution law

$$N = N_0 e^{-E/kt}$$

Consider two energy levels of an atomic systems E_1 & E_2 such that $E_2 > E_1$. Let N_1 & N_2 be the number of atoms per unit volume present at E_1 & E_2 respectively. Then by the Boltzmann's Distribution law,

$$N_1 = N_0 e^{-E/kt}$$

$$N_2 = N_0 e^{-E/kt}$$

$$\frac{N_1}{N_2} = e^{h\upsilon/kt}$$

Where K is the Boltzmann constant & $E_1 \sim E_2 = h\upsilon$

We know that when an assembly of atoms is exposed to light photon of energy $E_1 \sim E_2 = h\upsilon$, then the following three transitions takes place:

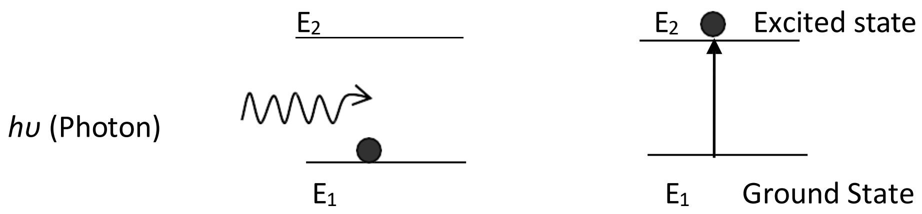
<u>Process 1</u> Stimulated absorption

Atoms in the ground state (E_1) are raised to excited state (E_2) after the absorbing a photon of energy $h\upsilon$, provided the photon energy is equal to the energy difference ($E_2 \sim E_1$). This is upward transition number of transition (Nab) occurring per unit volume per unit time is given by

$$(N_{ab}) = B_{12}\ N_1\ Q$$

Where B_{12} - probability of absorption transition N_1 – Number of atoms in state E_1

Q – Energy density of incident radiation



<u>Process 2</u> Spontaneous emission

The atoms in excited state E2 make a spontaneous transition to the ground state E1. This is a downward transition

Number of transitions occurring per unit volume per unit time $\mathbf{(N_{sp}) = A_{21}\ N_2}$

A_{21} - Probability of spontaneous transition from E_2 to E_1

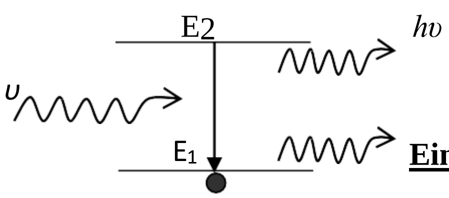
Number of atoms lying in the energy state E_2

<u>Process 3</u> Stimulated emission

The atoms in the excited state (E_2) may be forced to go to ground state (E_1) by striking the atom with a photon of energy $h\upsilon$, this is known as stimulated emission. It is downward transition

Number of transitions per unit volume per unit time $\mathbf{(N_{st}) = B_{21}\ N_2\ Q}$

B_{21} – Probability of stimulated transition



<u>Einstein Coefficients A & B:</u>

Under Equilibrium condition,

<u>No.</u> of upward transition per unit volume per unit time = <u>No.</u> of downward transition per unit volume per unit time

$$\mathbf{A_{21}\, N_2 + B_{21}\, N_2\, Q \quad = \quad B_{12}\, N_1\, Q}$$

(or) $B_{12} N_1 Q - B_{21} N_2 Q = A_{21} N_2$

(or) $Q (B_{12} N_1 - B_{21} N_2) = A_{21} N_2$

$$Q = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

$$Q = \frac{\frac{A_{21} N_2}{B_{21} N_2}}{\frac{B_{12} N_1}{B_{21} N_2} - 1}$$

We know that Planck's radiation for energy distribution in terms of frequency is

$$Q = \frac{8\pi h v^3}{e^{hv/kt} - 1}$$

$$\mathbf{B_{12} = B_{21}}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{c^3}$$

RESULT

(i) Equation states that stimulated emission rate per atom = stimulated absorption rate per atom.

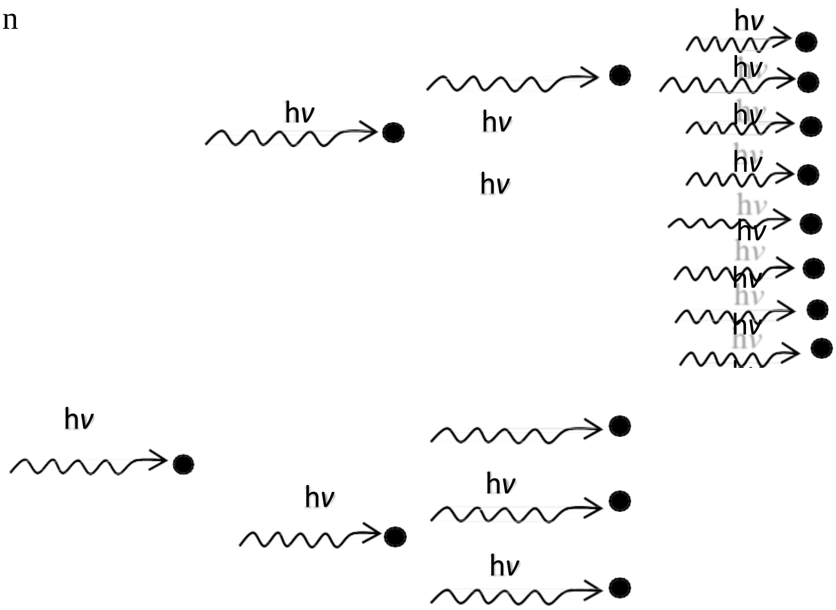
(ii) Equation shows that is proportional to frequency υ. The probability of spontaneous emission rapidly increases with the energy difference between the two states.

(iii) To achieve laser action, the stimulated emission should be predominant that spontaneous emission. To achieve this population inversion is required.

Concept of Laser

The photon emitted during stimulated emission has the same energy, phase, frequency and direction as that of the incident photon. Thus, we have two coherent photons in the above case. These photons now incident on two other atoms in the state E_2. This will result in induced emission of two more photons. Now there are four coherent photons of same energy. These four photons may induce further transitions with four other atoms in the energy state E_2. This gives rise to stimulated emission of eight coherent photons of same energy.

If the process continues in a chain, we will ultimately be able to increase the intensity of coherent radiation enormously. Stimulated emission is multiplied through a chain reaction. The multiplication of photons through stimulated emission leads to coherent, powerful, monochromatic, collimated beam of light. This light is known as LASER. Laser requires stimulated emission exclusively. This can be achieved by population inversion



Population inversion

Population inversion creates a situation in which the number of atoms in higher energy state is more than that in lower energy state. Usually at thermal equilibrium, the number of the atoms in the higher energy state N_2 is much smaller than the population of atoms at lower energy state N_1. i.e., $N_1 > N_2$.

The phenomenon of making the number of the atoms in the higher energy state greater than that of the lower energy state is called population inversion.

Condition

- There must be two energy levels
- There must be a source to supply the energy
- The atoms should be continuously raise to excited state

Active medium

A medium in which population inversion can be achieved is known as active medium.

Pumping Action

The process to achieve the population inversion in the medium is called pumping action.

Methods

(1) Optical pumping (excitation by photons)

(2) Electrical discharge method (excitation by electrons)

(3) Direct conversion
(4) Inelastic collision between atoms

(1) Optical pumping

When the atoms are exposed to light radiations of energy $h\nu$, atoms in the lower energy state absorbs these radiations and they go to the excited state. This method is called Optical pumping.

Eg: Nd - YAG Laser, Ruby laser

(2) Electrical discharge method

In this method, the electrons are produced in an electrical discharge tube. These electrons are produced in an electrical discharge tube. These electrons are accelerated to high velocities by a strong electrical field. These accelerated electrons collide with the gas atom.

By this process, energy from the electrons is transferred to gas atoms. Some atoms gain energy and they go to excited state. This results in population inversion. This method is called electrical discharge method.

$$A + e^* \rightarrow A^* + e$$

A – gas atom in ground state

e^* - Electron with high kinetic energy

A^* - same gas atom in excited state

E – Electron with lesser kinetic energy

(3) Direct Conversion

In this method, due to electrical energy applied in direct band gap semiconductor like GaAs, recombination process, the electrical energy is directly converted into light energy.

(4) Inelastic atom – atom collision

In this method, a combination of two gases is used say A & B. The excited states of A & B nearly coincide in energy. During electrical discharge atoms of gas A are excited to metastable state A* due to collision with electrons.

$$A + e^{*} \rightarrow A^{*} + e$$

Now, A* atoms at metastable state collide with B atoms in the lower state. Due to this inelastic collision B atoms gain energy and they are excited to a higher state (B*). Hence , A atoms lose energy and return to lower state

$$A^{*} + B \rightarrow A + B^{*}$$

This result is population inversion in the energy states of B.

Basic components of laser system

A laser system consists of three important components. They are

(a) Active medium (or) active material

(b) Pumping source

(c) Optical resonator

(a) Active medium (or) Active material

It is a medium in which atomic transitions takes place to produce laser action. The active medium may be solid, liquid, gas, dye or semiconductor.

(b) Pumping source

It is a system used to produce population inversion in the active medium.

(c) Optical resonator

An optical resonator consists of a pair of reflecting surfaces in which one is fully reflecting (R_1) and the other is partially reflecting (R_2). The active medium is placed in between these two reflecting surfaces

The photons generated due to stimulated emission are bounced back and forth between these two reflecting surfaces. This induces more and more stimulated transition leading to laser action.

Types of laser

Based on the type of active medium, laser systems are broadly classified as follows:

Sl.No	**Type of laser**	**Examples**
01.	Solid state laser	Ruby laser, Nd-YAG laser
02.	Gas laser	He – Ne Laser,Co_2 laser, Ar- ion laser
03.	Liquid laser	$SeOCl_2$ laser.

04.	Dye Laser	Rhodamine 6G laser
05.	Semiconductor laser	GaAs Laser, GaAsP laser

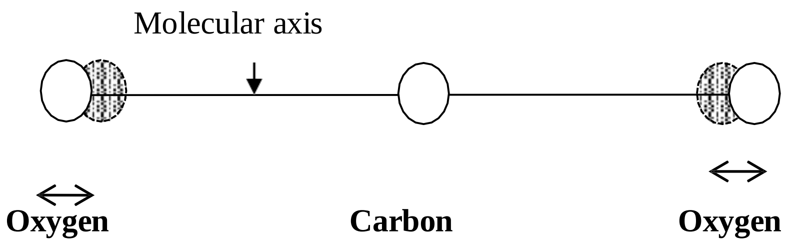
CO_2 laser

Modes of vibrations of CO_2 molecules:

A carbon dioxide molecule has a carbon atom at the centre with two oxygen atoms at extreme ends. This molecule exhibits three independent modes of vibrations

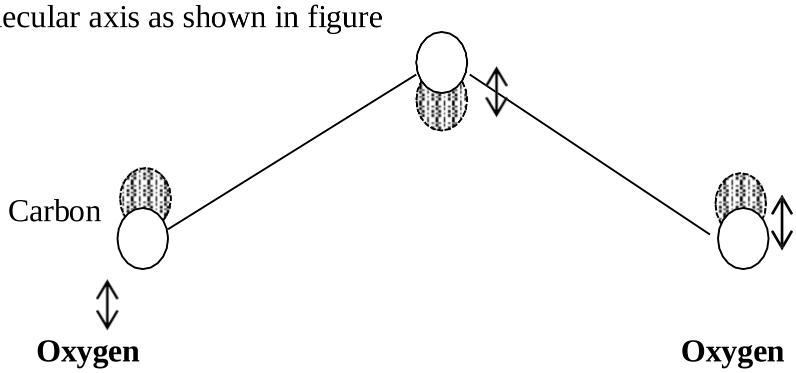
(i) **Symmetric stretching mode:**

In this mode of vibration, carbon atom is at rest in its position and both oxygen atoms vibrate along the molecular axis as shown in figure



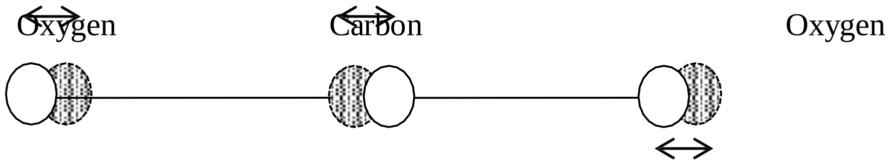
(ii) **Bending mode:**

In this mode of vibration, oxygen atoms and carbon atoms vibrate perpendicular to molecular axis as shown in figure



(iii) **Asymmetric stretching mode:**

In this mode of vibration, two oxygen atoms and central carbon atom vibrate asymmetrically i.e., both the oxygen atoms move in one direction while the carbon atom move in opposite direction as shown in figure

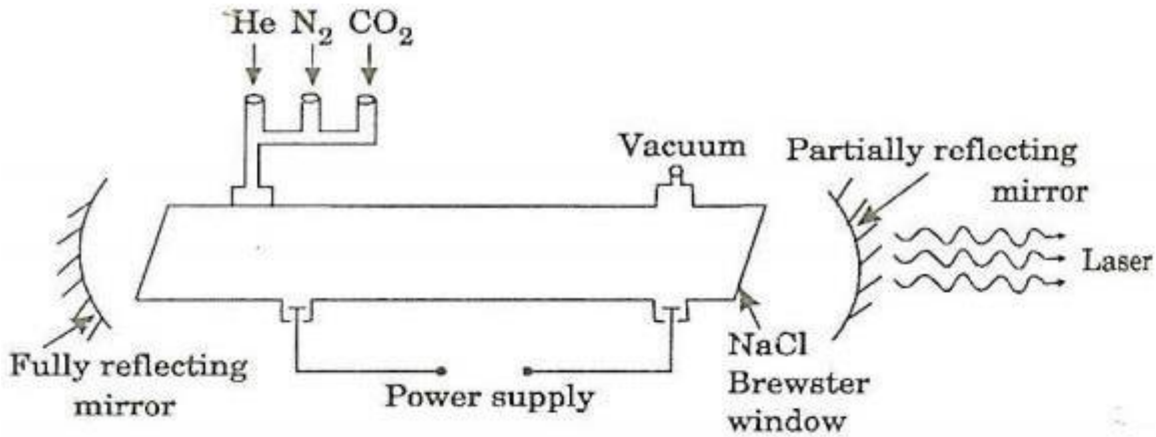


Principle:

The active medium is a gas mixture of CO_2, N_2 & He. The laser transition takes place between the vibrational states of CO_2 molecules

Construction:

The CO_2 laser consists of a quartz discharge tube of length 5m and diameter 2.5cm.The tube is filled with CO_2 along with nitrogen and helium gases with suitable partial pressures. The terminals of the discharge tube are connected to a D.C. power supply. The ends of the discharge tube are fitted with NaCl Brewster windows so that the laser light generated will be polarized. Two concave mirrors, one completely reflecting & the other partially reflecting act as optical resonator.



Working:

Figure shows the energy levels of nitrogen and carbon dioxide molecules. When an electrical discharge occurs in the gas, the electrons collide with nitrogen molecules and they are raised to excited states. This process is represented by the equation

$$N_2 + e^* \rightarrow N_2^* + e$$

N_2 – Nitrogen molecule in ground state

e^* - Electron with kinetic energy

N_2 – Nitrogen molecule in excited state

e – Same electron with less energy

Now, Nitrogen molecules in excited state collide with CO_2 atoms in ground state and excite them to higher electronic, vibrational and rotational levels. This process is represented by the equation

$$N_2^{*} + CO_2 \rightarrow CO_2^{*} + N_2$$

N_2^{*} – Nitrogen molecule in excited state
CO_2 - carbon dioxide molecule in ground state
CO_2^{*} - carbon dioxide molecule in excited state
N_2 – Nitrogen molecule in ground state

Since the excited level of nitrogen is very close to E_5 level of CO_2 atom, population in E_5 level increases.

As soon as population inversion is reached, any of the spontaneously emitted photon will trigger laser action in the tube. There are two possible transitions:

(i) Transition from $E_5 - E_4$ will produce a laser beam of wavelength 10.6µm
(ii) Transition from $E_5 - E_3$ will produce a laser beam of wavelength 9.6µm out of which 10.6µm is more intense than other.

Characteristics:

01.	Type	Molecular gas laser
02.	Active Medium	CO_2, N_2 & He
03.	Pumping Method	Electrical discharge
05.	Optical resonator	Two concave mirror from a resonant cavity
06.	Power output	10kW
07.	Nature of Output	Pulsed (or) Continuous beam of light
08.	Wavelength	9.6 µm &10.6 µm

Advantages:

(i) The construction is simple & output of the laser is continuous

(ii) It has high efficiency and high output power

(iii) This power can be increased by extend the length of the gas tube

Disadvantages:

(i) The contamination of oxygen by carbon monoxide will affect the laser action

(ii) Corrosion problem may occur in the reflecting mirrors

(iii) Accidental exposure will damage our eyes

(iv) The operation temperature plays a vital role for determining the output power

Applications:

(i) High power CO_2 laser used in material processing, welding, cutting, drilling, soldering , etc.,

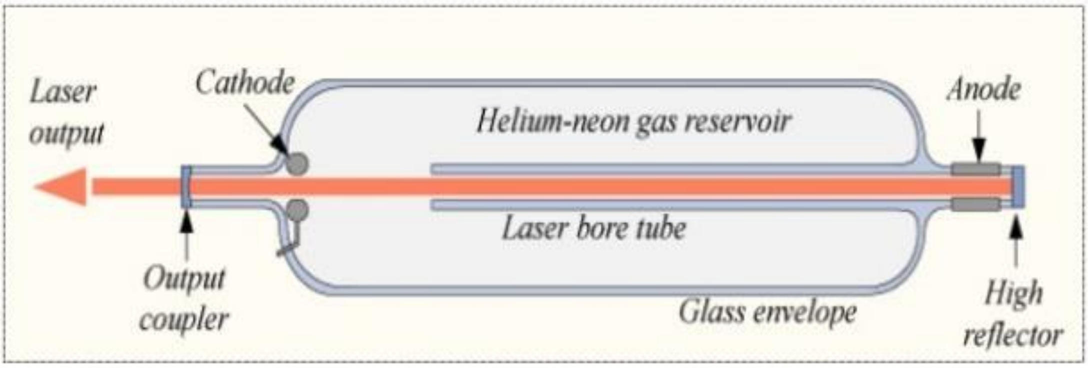
(ii) It is widely used in open air communication

(iii) It is used in remote sensing

(iv) It is used in the treatment of liver and lung disease

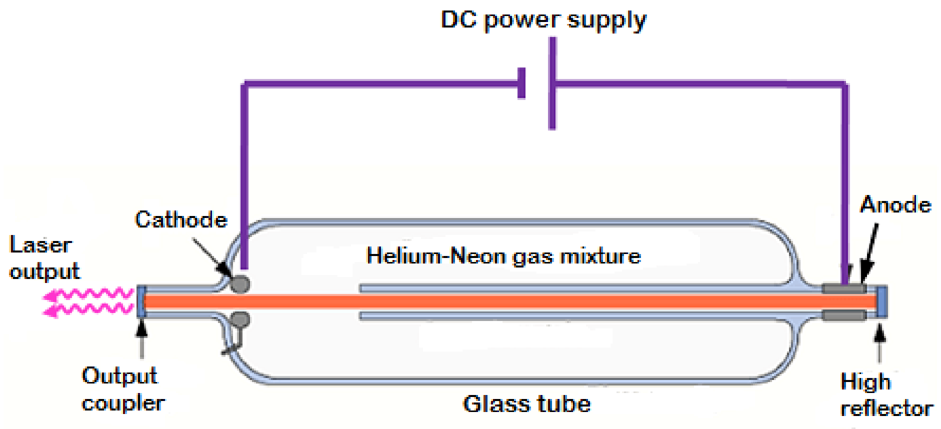
(v) It is used for bloodless operation and general surgery

HELIUM-NEON LASER



The helium-neon laser was the first continuous wave (CW) laser ever constructed

- The excitation of electrons in the He-Ne gas active medium is achieved by passing an electric current through the gas.
- The helium-neon laser operates at a wavelength of 632.8 nanometers (nm), in the red portion of the visible spectrum.



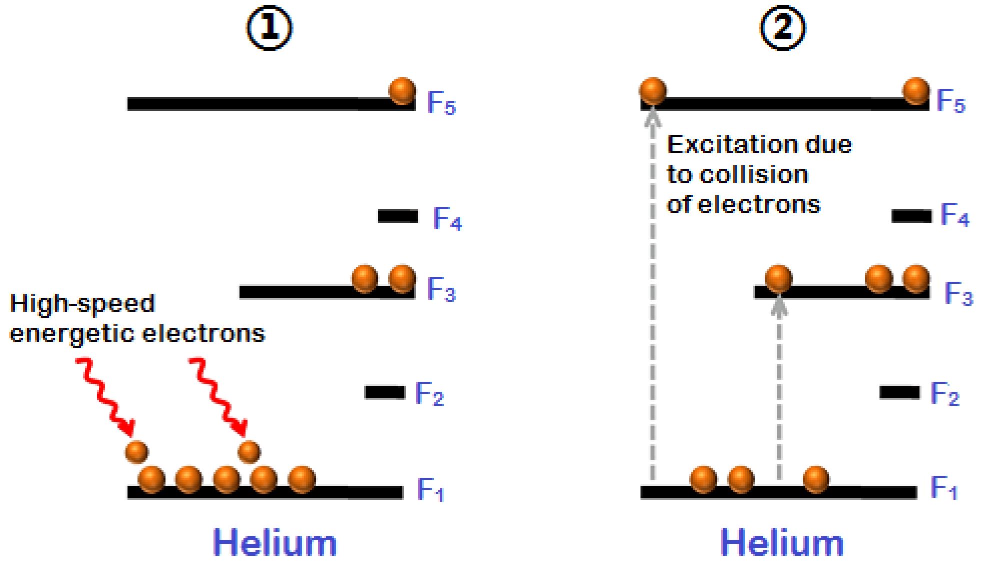
Helium-neon laser construction

The helium-neon laser consists of three essential components:

1. Pump source (high voltage power supply)
2. Gain medium (laser glass tube or discharge glass tube)
3. Resonating cavity
4. High voltage power supply
5. In helium-neon lasers, a high voltage DC power supply is used as the pump source. A high voltage DC supplies electric current through the gas mixture of helium and neon.

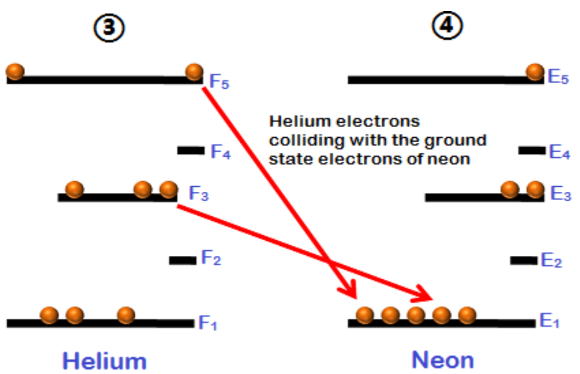
Gain medium

- ✓ The partial pressure of helium is 1 mbar whereas that of neon is 0.1 mbar.
- ✓ to excite primarily the lower energy state electrons of the helium atoms.
- ✓ neon atoms are the active centers and have energy levels suitable for laser transitions while helium atoms help in exciting neon atoms.
- ✓ Electrodes (anode and cathode) are provided in the glass tube to send the electric current through the gas mixture. These electrodes are connected to a DC

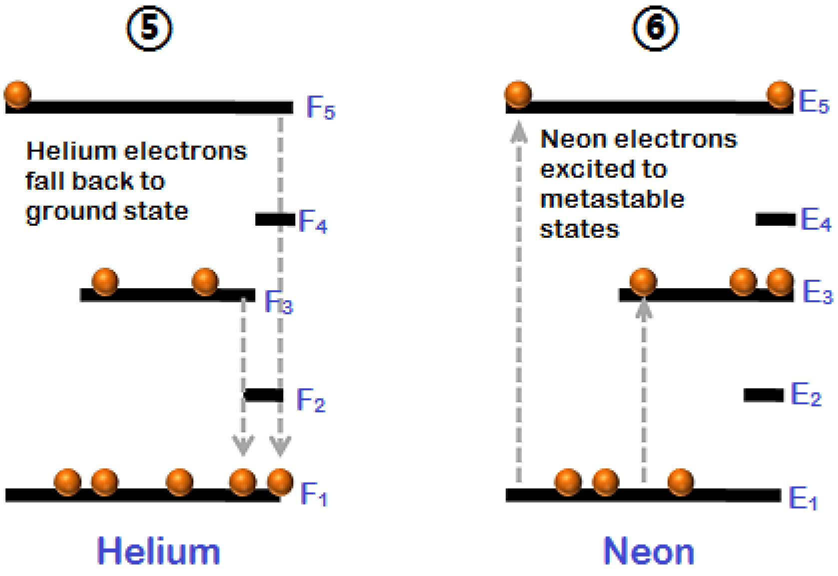


the power is switched on, a high voltage of about 10 kV is applied It is enough to excite the electrons and are accelerated

- Electrons transfer some of their energy to the helium atoms, jumps into the excited states
- Assume that these metastable states are F_3 and F_5
- Metastable state electrons of the helium atoms, return to ground state by transferring their energy to the lower energy state electrons of the neon atoms.
- The energy levels of some of the excited states of the neon atoms are identical to the energy levels of metastable states of the helium atoms.
- Let us assume that these identical energy states are $F_3 = E_3$ and $F_5 = E_5$. E_3 and E_5 are excited states or metastable states of neon atoms.



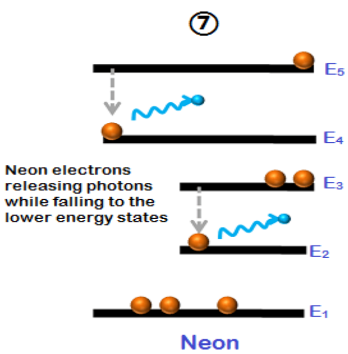
the lower energy state electrons of the neon atoms gain enough energy from the helium atoms and jumps into the higher energy states or metastable states (E_3 and E_5) whereas the excited electrons of the helium atoms will fall into the ground state. Thus, helium atoms help neon atoms in achieving population inversion.



millions of ground state electrons of neon atoms are excited to the metastable states having longer lifetime electrons (E_3 and E_5) of the neon atoms will spontaneously fall into the next lower energy states (E_2 and E_4) by releasing photons or red light.

Neon excited electrons continue on to the ground state through radiative and nonradiative transitions.

Photons emitted from the neon atoms will moves back and forth between two mirrors until it stimulates other electrons optical gain is achieved due to stimulated emission



photons emitted will escape through the partially reflecting mirror or output coupler to produce laser.

Advantages of helium-neon laser

1. Helium-neon laser emits laser light in the visible portion of the spectrum.
2. High stability
3. Low cost
4. Operates without damage at higher temperatures Disadvantages of helium-neon laser

5. Low efficiency
6. Low gain
7. Helium-neon lasers are limited to low power tasks

Applications of helium-neon lasers

1. Helium-neon lasers are used in industries.
2. Helium-neon lasers are used in scientific instruments.
3. Helium-neon lasers are used in the college laboratories

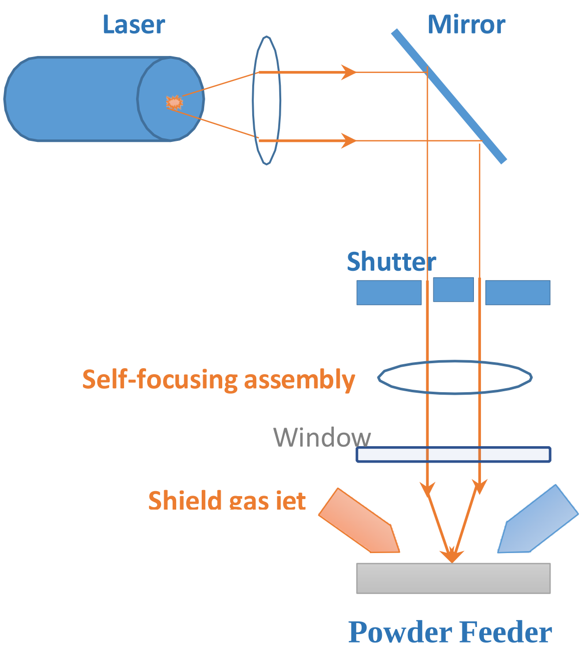
Basic applications of laser in industry

Material Processing:

Instrumentation

Construction

It consists of a laser source to produce laser beam, shutter to control the intensity of the laser beam and an assembly of lenses to effectively focus the laser onto the specimen. A shield gas jet is applied at one end of the surface of specimen to remove molten materials, smoke, fumes, etc., the powder feeder is placed at the other end of the surface of the specimen and feed the metal powder whenever necessary



Working

Laser source is switched ON. The light reflected from plane mirror is made to flow effectively on the surface of specimen with the help of self-focusing assembly. The shutter is used to control the intensity of light falling towards specimen.

Now the specimens gets heated and giving rise to fumes, smokes and molten materials. These are removed immediately from blowing the assisting gas from shield gas jet. Thus the laser light falls continuously on the specimen thereby increases the rate of fall. This process is used in drilling and cutting the materials using laser.

The powder feeder is used to spray metal powder over the specimen during the self-focusing effect while the laser is used for alloying, cladding, moulding, welding, etc.,

Welding

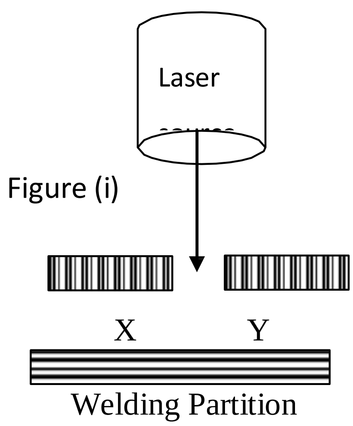
It is possible to use the CO_2 laser with increased power output as a welding tool. If a laser beam is focused on a particular area for a very long time, heating effect will produced in that area. This is known as thermal effect. This principle is employed in laser welding. From the figure (i), if we consider welding of two metal plates, say X and Y, which are held in contact at their edges and a laser beam is made to move along the line of contact of the plates. The laser beam heats the edges of the two plates to their melting points and causes them to fuse together.

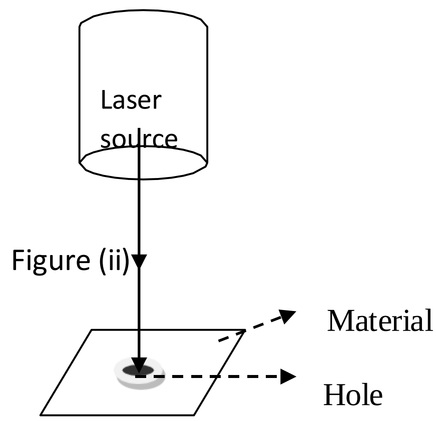
This welding is contactless, therefore there is no need for introducing deleterious impurities in the material

(i) Very high welding rates are possible

(ii) Dissimilar metals are welded

(iii) Complex contour can be welded





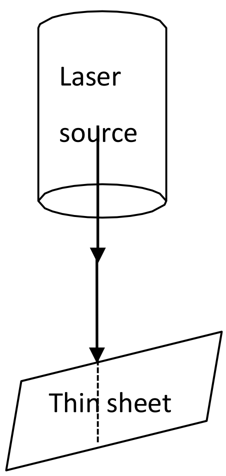
(ii) Heat treatment

Thermal effect is the basic principle used in laser heat treatment.. Nd - YAG and CO_2 lasers used for this purpose. The heating treatment is shown in figure (ii). When light energy is converted to heat energy, the material may be heated, then melted followed by vaporized. Laser heat treatment provides selective treatment of the desirable areas which are subjected to the more intense use. Hence it is used in melting materials and drilling holes

(iii) Laser Cutting

The principle behind cutting is the vaporization of the material at the focus of the beam. Any desire shape can be cut. The high intensity laser beam is made to fall on the material as shown in figure (iii). Due to the application of laser beam on the material for a long time, the temperature of the material increases. it can hence be used for cutting samples.

- The cutting used to be very smooth.
- Requiring no further treatment such as grinding & polishing,
- Possibility of precise shape cuts



(iv) Laser Drilling

This process is a non - contact and does not require a physical drill bit. Drilling holes by laser beam is based on the intense evaporation of material heated by a series of powerful light pulses of 10^{-4} to 10^{-3} s duration. A laser pulse having energy of about 0.05J and pulse width of 10^{-3} s can drill a hole of about 1Å radius in a steel plate of thickness 1mm.

(v) Soldering

It is a process in which two or more metals are joined together by melting and putting a filler material (solder) into the joint, the filler metal having a lower melting point than the adjoining metal.

Laser soldering, the newest soldering method.

Laser soldering

It is a process in which selectively heats solder by means of laser irradiation to form a bond between two parts.

Principle

Laser soldering is a technique where a precisely focused laser beam provides controlled heating of the solder alloy leading to a fast and non-destructive of an electrical joint.

The process uses a controlled laser beam to transfer energy to a soldering location. The absorbed energy heats the solder until it reaches its melting temperature leading to the soldering of the contact and this completely eliminates any mechanical contact.

Working

Laser soldering is a technique where a 30 – 50 W laser is sued to melt and solder an electrical connection joint. Diode laser system based on semiconductor junctions are used for this purpose.

The wavelength are typically 808 nm through 980 nm. The beam is delivered via an optical fiber to the work piece, with fiber diameters 800 mm and smaller.

Since the beam out of the end of the fiber diverges rapidly, lenses are used to create a suitable spot size on the work piece at a suitable working distance. A wire feeder is used to supply solder. Both lead – tin and silver – tin material can be soldered.

Laser soldering process

- The laser illuminates the soldering point.
- The illuminated area emits heat (surface heat emission)
- The heat transfers into the surrounding area and is raised to the melting temperature.
- Solder is supplied.

Types of laser used in soldering

Three main types of lasers are found suitable for soldering process. They are

1. Carbon di oxide laser
2. Nd:YAG laser
3. Semiconductor laser

Advantages of laser soldering:

In contrast to other conventional soldering techniques, laser soldering offers a lot of advantages. They include

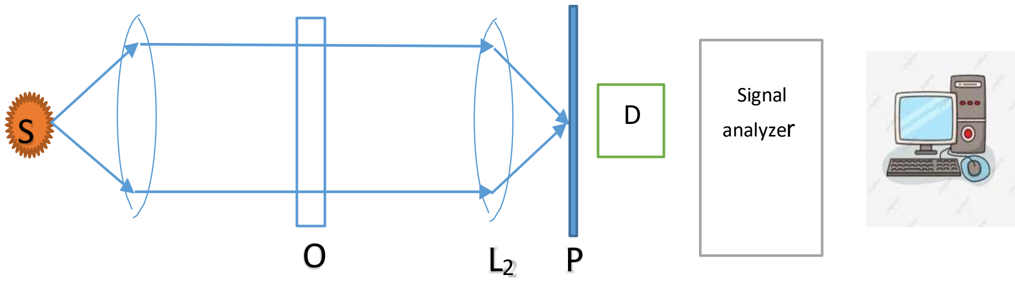
- Localised / selective heat input and Ideal for heat sensitive components.
- High precision – spot sizes in the order of 100s of microns
- Fast control of heat input (laser on / off)
- It reduces intermetallic compound formation and produce high quality joint.
- It also has low maintenance.

(vi) Surface Defect Detection

High intensity laser beam is used to study the surface defects in materials such as ICs, aircrafts, etc. The laser beam reflected from the surface of the material under investigation. The laser beam reflected from the surface of the material under investigation. The laser light is also directly obtained from the source as a reference are used to produce interference between the two laser beams. Information about the material is obtained by forming the interference pattern.

Figure shows the experimental arrangement to study surface defects in materials. A high intensity laser beam from the source S falls on the converging lens L_1. The lens L_1 focuses the laser beam on the object O. The optical diffraction pattern of the image is focused on the photographic plate (P) or photodiode (D).

The photodiode senses the light and converts the light energy into electrical signals. Using the signal analyzer along with the necessary software, the image of the defect is obtained. Thus surface defects in materials can be studied.



Holography

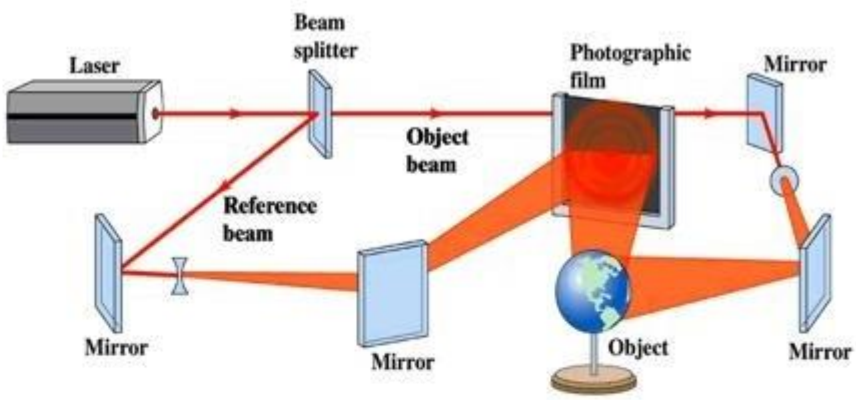
In conventional photography a negative is made first and using it a positive print is produced later. The positive print is only a 2D record of light intensity received from 3D object. It contains information about the square of the amplitude of the light wave that produce the image but information about the phase of the light wave is not recorded and is lost.

In 1947, Dennis Gabor recorded the phase and intensity components of the wave where 3D image of an object is recorded. This is named as Hologram.

Explanation

A weak but broad beam of laser is split into two beams: reference beam and object beam. The reference beam is allowed to reach the photographic plate directly while the object beam illuminates the object.

The part of the light is scattered by the object travel towards the photographic plate and interferes with the reference beam and produces an interference pattern on the photographic plate. The photographic plate carrying the interference pattern is called hologram. Here, Holo = complete (Greek); gram = writing.



(vii)laser in communication

1. Since the laser beam has enormous bandwidth and it permits 10 million telephone conversation (or) 8000 TV programmes simultaneously.
2. Narrow angular spread and directionality of laser beam makes it a very useful tool for communications with satellites and rockets to the moon and other planes (inter planetary communication)
3. The laser light is not absorbed by water and hence it is utilized in underwater communication between sub marines.

(viii) Laser in Engineering

1. The laser beam is highly energetic, hence it is used to destroy huge objects like aircrafts, missiles, etc., in few seconds by directing the laser beam on the objects. Hence it is also called as death ray of war weapon.
2. The laser gun is high convergent, hence focussing a spot at a short range vapourise the governing part of the object
3. Laser beam can exactly determine the size, form, distant, velocity and direction of any distant object (missile, war plane, etc.,) by receiving the reflected laser beam on a cathode screen as in RADARs.
4. It is used for automatically guiding rocket and satellite
5. It is used for forecasting earthquakes
6. It is used to take printouts